

Load-Spread Angles for Circular Footing on Two-Layered Soil System

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ABSTRACT

The paper presents an analysis of a uniformly loaded circular footing on two-layered soil system consisting of stiff layer overlying soft one. The variation of normal stresses with distance at the interface between the two layers is obtained using PLAXIS software for different modular (E_1/E_2) and relative thickness (h/a) ratios where E_1 and E_2 are deformation moduli of the stiff upper and lower layers respectively, h – the thickness of the upper layer and a – the radius of the footing. A Bell-curve is fitted to this variation of normal stresses with distance and integrated to estimate radial extents for different percentages of load transferred at the interface. The load spread angle defined as the ratio of extent of radial distance over which a prescribed percentage of loads is transferred to the thickness of the upper layer is obtained for different modular and relative thickness ratios and for different percentages of load. Load spread angle increases with increase in modular ratio, E_1/E_2 and with h/a . The load spread angle increases marginally for thickness ratio, h/a , greater than 2.0.

KEYWORDS: Homogenous Soil Bed, Two-Layered System, Normal Stresses At Interface, Load Spread Angle, Modular Ratio.

MATERIALS & METHODS

Axis symmetric finite element analysis with PLAXIS is carried out to obtain the vertical stress distribution at the interface of two-layered soil and at different depths for homogenous layer. PLAXIS 2D an FE program specially developed for geotechnical engineering is adopted. Two cases are analyzed, viz., circular footing on (i) homogeneous bed and (ii) two layered system.

PROBLEM DEFINITION AND ANALYSIS

A uniformly loaded circular footing of diameter (width), $B (=2a)$ or of radius, a , with uniformly applied load of q , on a two-layered soil is considered (Fig.1). The thickness of the top layer is h while the bottom layer extends to infinity. The deformation moduli and Poissons ratios of the two layers are E_1 & ν_1 and E_2 & ν_2 respectively. The problem being axi-symmetric, only half of the region is considered for the analysis⁴.

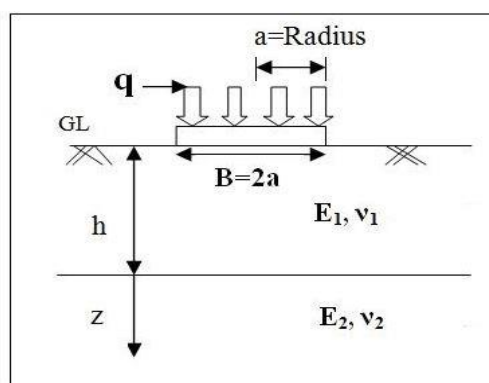


Fig.1 Definition sketch-Uniformly loaded circular footing on two-layered soil

Modular ratio, μ , ($E_1/E_2=1$) represents homogenous soil while $\mu>1$ represents a stiff (reinforced) foundation bed on soft deposit. Normal stresses, σ_z , normalized with applied stress³, q , along horizontal planes at normalized depths, z/a , equal to 0.2 to 2.0 are evaluated and plotted against radial distance for homogeneous ground (Fig. 2). The stress beneath the center of the footing decreases from near 100% for $z/a = 0.2$ to about 30% for $z/a = 2.0$. Correspondingly normal stresses decrease away from the center of the footing and are more uniform at deeper depths than at shallow ones attesting to the fact that stresses tend to become more uniform with depth.

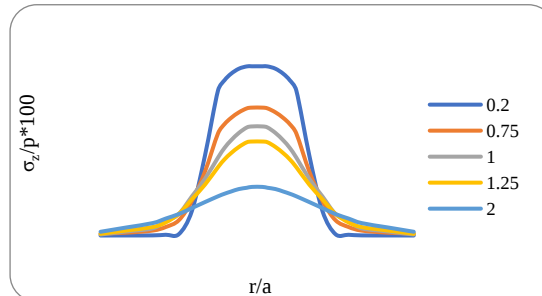


Fig. 2 Vertical stresses at different depths for homogeneous ground

The variations of normal stresses along the interface of two-layered soil, with respect to r/a for modular ratios $\mu = 2, 3, 6, 10$ and 20 for relative thickness ratios, h/a , equal to 0.2 are presented in Fig.3. For $h/a=0.2$ (Fig. 3), the normalized stresses below the center of the footing are 58% and 16% for modular ratio, $\mu = 2$ and 20 respectively. i.e., the normalized stress below the center decreases significantly with increasing ' μ '. Similar results are shown for relative thickness ratios of $0.75, 1.0, 1.25$ and 2.0 respectively (Figs. 4 to 7).

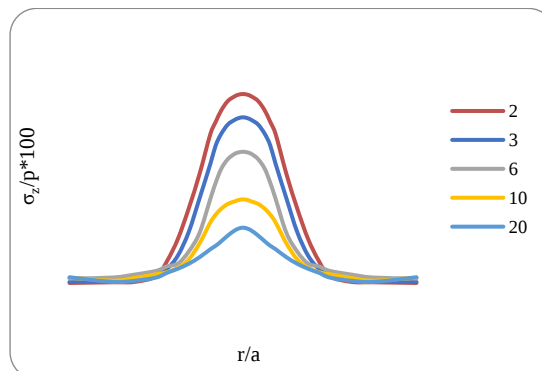


Fig. 3 Normalized stresses along the interface -h/a=0.2

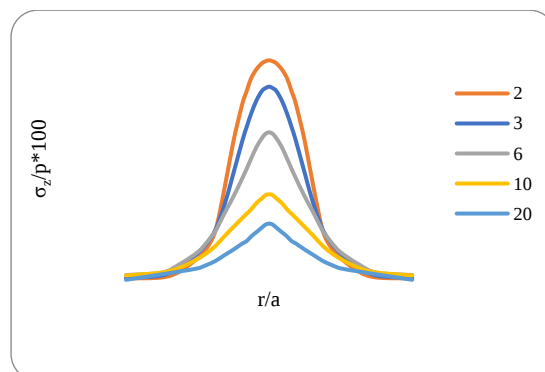


Fig. 4 Normalized Stresses along the Interface- h/a=0.75

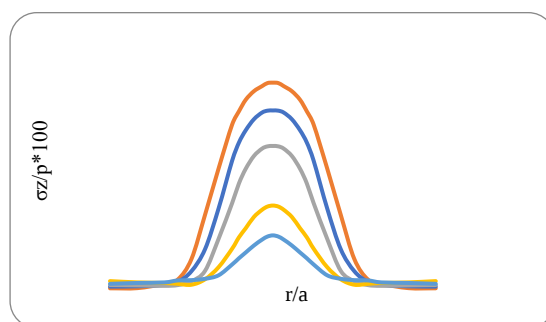


Fig. 5 Normalized Stresses along the Interface - $h/a=1.0$

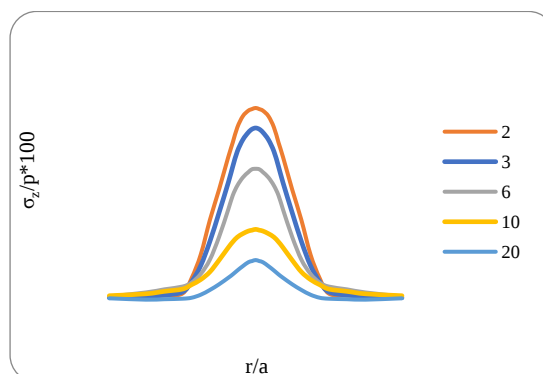


Fig. 6 Normalized Stresses along the Interface- $h/a=1.25$

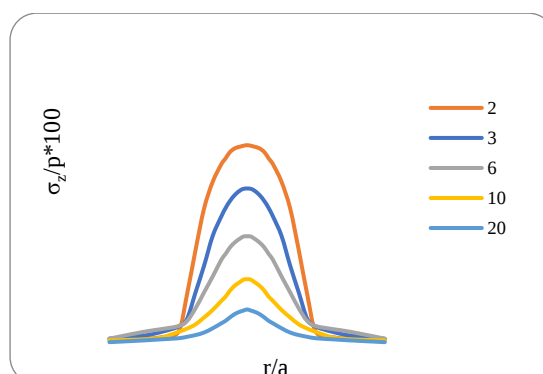


Fig. 7 Normalized Stresses along the Interface - $h/a=2.0$

DISTRIBUTION OF NORMAL STRESS WITH RADIAL DISTANCE

The variation of normal stress with radial distance , r , can be represented by a bell curve as depicted in Fig. 8.

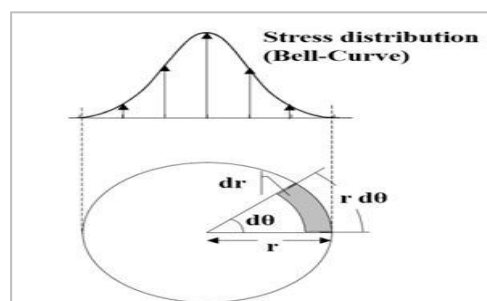


Fig. 8 Typical variation of normal stress with radial extent – the Bell curve

The expression for the variation of normal stress with distance, r , along the interface treated as Bell curve is

$$\sigma_z = ue^{-\left(\frac{(r-v)^2}{2w^2}\right)} \quad (1)$$

where u, v and w are the Gaussian coefficients and 'r' is the radial distance from the central axis of the footing. Constants 'u', 'v' and 'w' for the Bell curve are estimated using curve-fitting.

The total load, P_α , transmitted over a radius, R_α along the interface is

$$P_\alpha = \int_0^{R_\alpha} \int_0^{2\pi} \sigma_z r dr d\theta \quad (2)$$

where R_α is radial distance from the axis of the footing over which the desired percentage of load is transferred. Normalizing, P_α with the total applied load, $P_0 (= \pi a^2 q)$, one gets

$$\lambda_\alpha = \frac{P_\alpha}{P_0} = \frac{2 \int_0^{R_\alpha} u e^{-\left(\frac{(r-v)^2}{2w^2}\right)} \left(\frac{r}{a}\right) \left(\frac{dr}{a}\right)}{q} \quad (3)$$

where $\lambda_\alpha = P_\alpha/P_0$ – ratio of load transferred over the circular area with radius, R_α and the applied load, $P_0 = \pi a^2 q$.

Ratios, λ_α of load transferred over a circular area with radius, $R_\alpha (= \alpha_a)$, to the applied load are evaluated for modular ratios, μ equal to 1, 2, 3, 6, 10 and 20 and for $h/a = 0.2, 0.75, 1.0, 1.25$ and 2.0. ' μ ' of 1 corresponds to a homogenous layer while μ of 20 implies very stiff layer. Stiffer the upper layer, the better is the spread of the applied load over wider areas. Fig. 9 shows the effect of percentage of total load, λ_α with the extent of radial distance (R_α) for different thickness ratios, z/a for homogeneous layer ($\mu=1$). For 85% load transfer, the radial extent, R_α increases from 1.1 to 2.3 for z/a increasing from 0.2 to 2. Similarly for 95% load transfer the radial extent increases significantly from 1.2 for $z/a=0.2$ to 2.8 for $z/a = 2$ for homogeneous soil, μ (E_1/E_2) = 1.0.

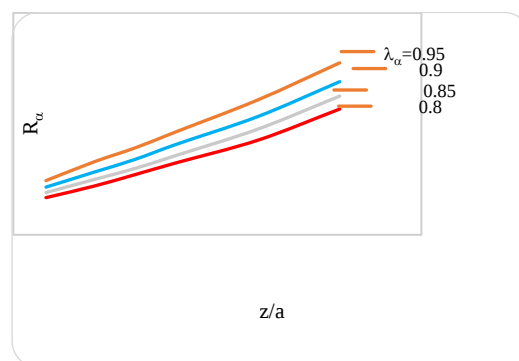


Fig. 9 R_α vs z/a - Effect of λ_α for homogeneous soil ($\mu=1$)

Radial extent, R_α is plotted against modular ratio, μ and λ_α for $h/a=0.2$, Radial extents are 1.3 and 2.2 corresponding to $\mu=2$ and 20 (Fig.10). For $h/a = 0.75$ (Fig.11) R_α increases from 1.8 to 3.8 for 85% load for μ increasing from 2 to 20. The load transferred at the interface increases to 95% for radial extent ranging from 1.4 for $\mu=2$ to 2.6 for $\mu=20$ for the relative thickness of layer, $h/a=0.2$. R_α increases from 2.1 to 4.1 for μ increasing from 2 to 20 for relative thickness of layer, $h/a = 0.75$ for 95% load. The variation of R_α with μ tends to linearity with increasing modular ratio and relative thickness of layer, $h/a = 0.2$ to 2.0. Similar plot is given for $h/a = 1.0$ where in the results are similar to those for $h/a=0.2$ & 0.75².

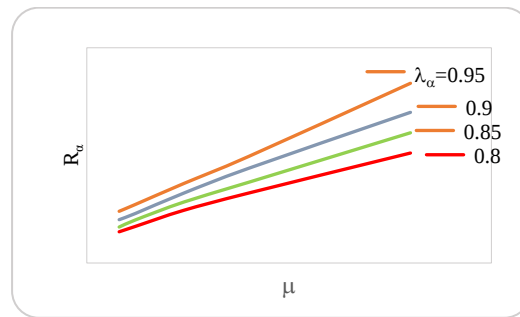


Fig. 10 R_α vs μ for $h/a = 0.2$

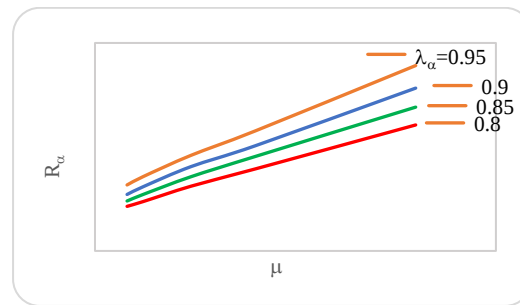


Fig. 11 R_α vs μ for $h/a = 0.75$

Fig. 12 presents the variation of radial extent R_α , with the modular ratio, μ for λ_α equal to 0.8, 0.85, 0.9 and 0.95. For 85% load transfer, the radial extent, R_α increases from 1.8 to 3.8 for modular ratio increasing from 2 to 20. Similarly for 95% load transfer the radial extent increases significantly from 2.0 for $\mu=2$ to 4.5 for $\mu = 20$ for relative thickness layer $h/a=1.0$.

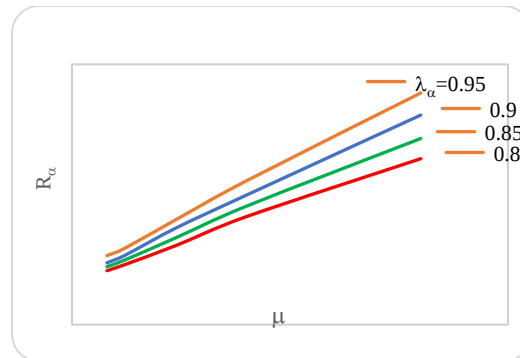


Fig. 12 R_α vs μ for $h/a = 1.0$

Contrastingly for 85% load transfer, the radial extent increases from 2.0 to 4.2 for modular ratio, μ increasing from 2 to 20. For 95% load transfer the radial extent increases significantly from 2.2 for $\mu=2$ to 5.1 for $\mu=20$ for relative thickness layer $h/a=1.25$ (Fig. 13).

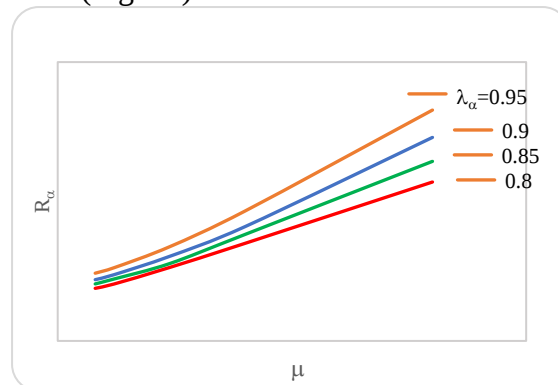


Fig. 13 R_α vs μ for $h/a = 1.25$

Fig. 14 presents the variation of extent R_α , with the modular ratio, μ for λ_α equal to 0.8, 0.85, and 0.9 and 0.95. For 85% load transfer, R_α extent increases from 2.4 to 4.5 for modular ratio, μ increasing from 2 to 20. For 95% load transfer R_α extent increases significantly from 2.6 for $\mu=2$ to 5.4 for $\mu=20$ for relative thickness layer $h/a=2.0$.

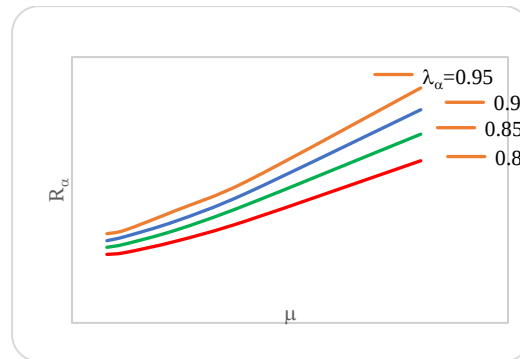


Fig. 14 R_α vs μ for $h/a = 2.0$

LOAD-SPREAD ANGLE (θ) FOR TWO LAYERED SYSTEM

Stiff upper layer redistributes the applied load over a wider area. As presented in Fig. 15, $(R_\alpha - a)$ is the extent beyond the edge of the footing over which the load gets distributed at the interface of the two layers due to the stiff layer effect. The load-spread angle, ' θ ' is given by

$$\theta = \tan^{-1} \left[\frac{(R_\alpha - a)}{h} \right] \quad (4)$$

where R_α – is radial distance from the axis of the footing over which the desired percentage of load is transferred..

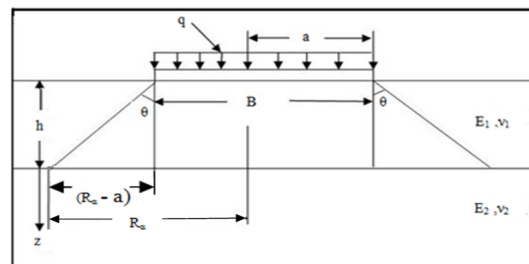


Fig. 15 Load spread angle for two layered system

The load spread angle, θ is determined for both homogeneous and two layered systems using Eq.4, for different percentages of load, λ_α of 80%, 85%, 90% and 95% and presented as function of z/a in Fig.16 for the homogeneous soil condition. ' θ ' is 9.1° for 80% load and increases gradually to 18.2° for 95% load for $z/a = 0.2$. For $z/a=2.0$, ' θ ' increases from 40.2° to 53.5° for 80 and 95% loads respectively¹.

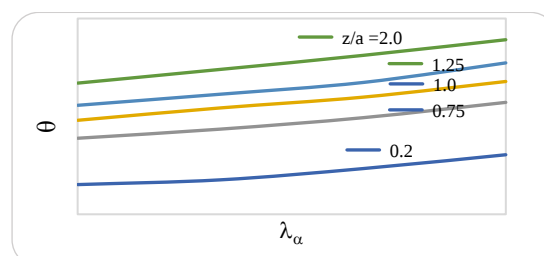


Fig. 16 θ vs λ_α for homogeneous soil $\mu = 1$

One can observe a moderate increase of ' θ ' with h/a corresponding to better spread of the load with increasing thickness of the stiff upper layer. Higher the stiffness ratio, μ , larger is the load spread angle (Fig.17).

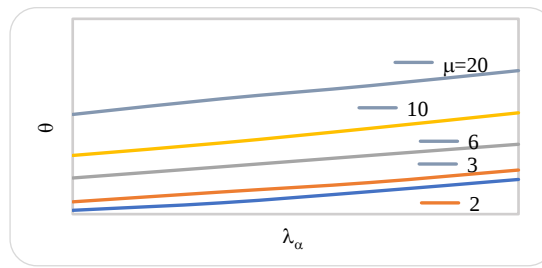


Fig. 17 θ vs λ_α for $h/a=0.2$

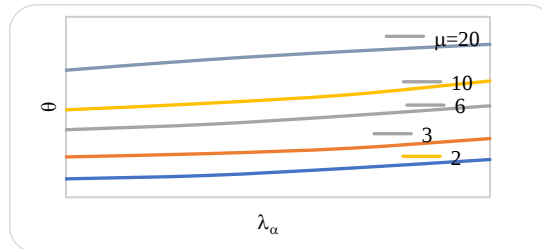


Fig. 18 θ vs λ_α for $h/a=0.75$

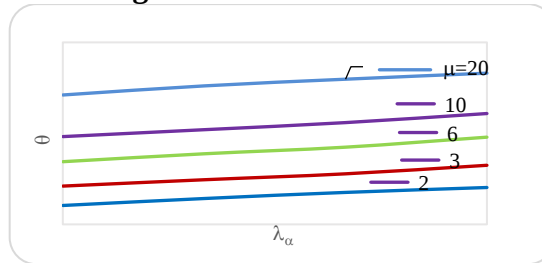


Fig. 19 θ vs λ_α for $h/a=1.0$

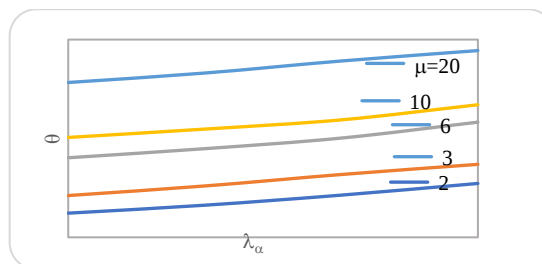


Fig. 20 θ vs λ_α for $h/a=1.25$

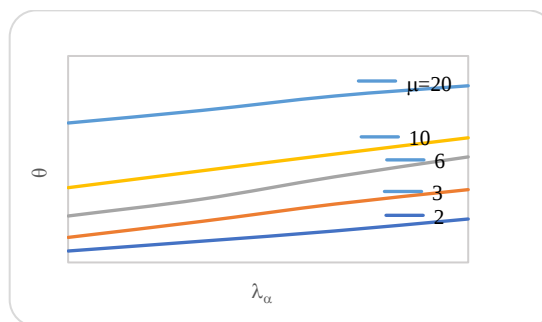


Fig. 21 θ vs λ_α for $h/a=2.0$

Variations of ' θ ' with ' λ_α ' with h/a are shown in Figs. 17 to 21 for h/a equal to 0.2 to 2.0. Load spread angle, ' θ ' increases with modular ratio, ' μ '. ' θ ' increases from 16.1° to 25.6° for $h/a=0.2$ for 80% to 95% of modular ratio, $\mu = 2$ (Fig. 17). ' θ ' increases gradually from 45.6° to 59.0° with percentage load, λ_α from 80% to 95% for $h/a = 0.2$ and for $\mu = 20$. Similar results are obtained for all thicknesses and modular ratios (Figs. 18 to 21).

Load spread angle (θ) vs h/a for $\lambda_\alpha = 90\%$

Load spread angles are calculated initially for relative thickness $es, h/a \leq 2$. A second degree polynomial equation of the form $\theta = C_1 + C_2(h/a) + C_3(h/a)^2$ is fitted to the θ values up to 2.0 for all the curves for $\mu=1$ to 20. Constants C_1 , C_2 and C_3 are evaluated for the respective μ from the known values of θ for h/a equal to 1.0, 1.5 and 2.0. Knowing the values of the constants C_1 , C_2 and C_3 , θ for h/a equal to 2.5, 3, 3.5 and 4.0 are then calculated and the load spread angle, θ , extrapolated for h/a values upto 4.0 for $\mu=1$ to 20⁶.

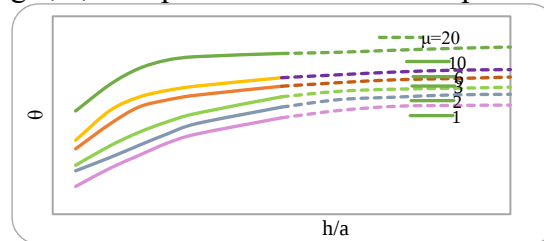


Fig. 22 θ vs h/a for $\lambda_\alpha = 90\%$

Fig. 22 presents the variation of load spread angle with respect to h/a as a function of modular ratio for $\lambda_\alpha = 90\%$. ' θ ' increases from 14.0° to 55.1° for homogeneous soil, ($\mu=1$), for h/a increasing from 0.2 to 4. ' θ ' increases from 52.1° to 84.5° for h/a increasing from 0.2 to 4 for modular ratio $\mu=20$. ' θ ' increases significantly with h/a for h/a equal to 2.0 and marginally for $h/a > 2.5$.

CONCLUSIONS

On the basis of the analysis using PLAXIS of a circular footing on a homogeneous and two-layered soil, the following conclusions are drawn:

- The normalized stresses beneath the centre of the footing decrease significantly with depth in homogenous ground and with increasing modular and thickness ratios in case of two-layer soils.
- The radial extent, R_α over which a given percentage of load, λ_α gets spread increases with increasing relative thickness, h/a , and modular, μ ratios⁵.
- The load spread angle, ' θ ' increases with increasing relative thickness, h/a , and modular, μ , ratios.
- A plot is developed for the variation of load spread angle, ' θ ' with relative thickness ratio, h/a , for a range of modular ratios, μ .

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