ISSN: 2178-7727

Dynamic Nature of Cracked Beams : A Mathematical Analysis

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ABSTRACT

we develop a mathematical analysis for dynamic nature of cracked beam under the assumption of strong and weak dampness by using first principles in terms of sub-differentials of the bending and axial potential energies.

KEYWORDS: Cracked beam equation of motion, sub- differential, principle of strong damping, weak damping.

INTRODUCTION

The main objective of this paper to analyze dynamic nature of cracked beam and shall arches based on mathematical frame-work, we derive equation of motion and establish existence and uniqueness of result for such equation. The presence of cracks tends to weaker regularity which results in arches motion [7, 8].
Let the transverse motion of beam is given by function $x(y,t) and\ y \in (0,\tau), t \geq 0$.

Which shows deformation of beam from x-axis boundary conditions are

$$x(0,t) = x''(0,t) = 0,$$

$$x(\tau,t) = x''(\tau,t) = 0, t\epsilon[0,T] \dots\dots (1)$$

We assume these are 'n' cracks in the beam at the points

$$0 < y_1 < \cdots \dots \dots < y_n \dots < \tau.$$

Now we define special Hilbert space which is broad enough to have continuous functions and discontinuous derivative at joint crack points such that

$$U \subset H_0' \subset H \subset (H_0')' \subset U'$$

Now we define operator $F: U \to U'$

And

$$F(u,v)_v = \sum_{i=1}^{n+1} (u'',v'') + \sum_{i=1}^{n} \frac{1}{\phi_i} J(v'')(y_i), J(u')y_i \dots\dots\dots\dots (2)$$

$for\ u,v\ \epsilon\ U\ and\ \phi_i\ is\ the\ discontinuity\ at\ i^{th}\ crack$

$$J(v')(y) = v'y^+ - v'y^-$$

J is the Jump point between two cracks. For substantial literature survey and reviews of elements with cracks you can refer [1-6] and [9-14].

ACTA SCIENTIAE
ISSN: 2178-7727

Hilbert Space

In this section we introduce Hilbert space H, V, H_0^1 which is much suitable for cracked elements see [15]. Suppose that arch has n cracks at the points $0 < y_1 < y_2 < \cdots < y_n < \tau$ in the interval $[0, \tau]$. $now\ any\ subinterval\ l_i = (y_{i-1}, y_i)\ , i = 1,2,3 \ldots., n+1\ is\ defined$ in the interval $[0, \tau]$.

Let H be the Hilbert Space

$$H = \oplus_{i=1}^{n+1} L^2(l_i) \ldots \ldots \ldots \ldots \ldots (3)$$

We denote inner product by$(.\ ,\ .) and\ norm\ in\ L^2(l_i)\ \ , i = 1,2,3 \ldots., n+1$ by$|.|$, which is defined as$(u, v)_H = \sum_{i=1}^{n+1}(u, v)$,

$$|u|^2{}_H = \sum_{i=1}^{n+1} |u|^2{}_i \ldots \ldots \ldots \ldots \ldots (4)$$

Let us defined the linear space

$$V = (v\epsilon \oplus_{i=1}^{n+1} H^2(l_i): V(0) = V(\tau) = 0, J(V)y_i = 0; i = 1,2..n \ldots \ldots (5)$$

The inner product on V is given by

$$(u, v)_V = \sum_{i=1}^{n+1} (u'', v'')_i + \sum_{i=1}^{n} J(u')(y_i)\ for\ u, v \in V \ldots \ldots (6)$$

Where $(u'', v'')_i = \int_{r_i} u''(y)v''(y)dy$

The associated norm V is defined in $L^2(l_i)$ as follows:

$$||v||^2{}_V = \sum_{i=1}^{n+1} |v''|^2{}_i + \sum_{i=1}^{n} |J[v']y_i|^2\ , for\ any\ v\epsilon V \ldots \ldots (7)$$

The derivatives of u and v are defined component wise in the space $H^2(l_i)$ and they are continuous in the interval$[0, \tau]$.

VARIATIONAL SETTING FOR OPERATOR *F*

Lemma –Let F be defined by equation (2) then$F$is continuous, Linear and symmetric and coercive operator since all $\phi_i \geq 0$.

Now we introduce our boundary conditions as,

$$v(0) = v(\tau) = 0, v''(0) = v''(\tau) = 0 \ldots \ldots \ldots \ldots \ldots .. (8)$$

$$and\ J(v)(y_i) = 0, J(v'')(y_i) = 0, J(v''')(y_i) = 0$$

$$J(v')(y_i) = \emptyset_i J(v'')(y_i^+)$$

For $i = 1,2,3 \ldots n,$ ... ... (9)

$J\ is\ bounded\ in\ J(0, \tau)$.

MAIN RESULT

Beam equations of motion

After variational formulation in this section we derive beam equation of motion by using extended Hamilton principle, here we have two types of potential energies $V_a(v)$, due to axial force and $V_b(v)$, due to bending

Now sub-differential is calculated by

$$\partial\varphi(v) = Fu = -\sum_{i=1}^{n} J(v')(y_i)\delta(y - y_i) - v'' \ldots \ldots \ldots (10)$$

Then sub-differential of bending $\partial\, V_b(v) = Fu$.

And we know that

$$x + \partial V_b(x) + \partial V_a(x) + C_d(x) = p \ldots \ldots \ldots \ldots \ldots \ldots . \quad (11)$$

ISSN: 2178-7727

Is governing abstract equation of beam C_d is damping coefficient and p is the function of y and t, then

$$x + F(y) - \frac{1}{\tau}(\alpha + \frac{1}{2}|x'|^2{}_H)(\sum J(x')(y_i)\delta(y - y_i) - x'') + C_d(x) = p \ldots\ldots\ldots (12)$$

Is abstract equation of beam with cracks, now by applying our boundary conditions we have:-

$$x + x'''' - \frac{1}{\tau}(\alpha + \frac{1}{2}|x'|^2{}_H)(\sum_{i=1}^{n} \phi x''(y_{i,}t)\delta(y - y_i) - x'') + C_d(x) = p \ldots\ldots\ldots (13)$$

It is called classical equation of beam, this is also refer the case of weak damping where dynamic viscosity coefficient $\mu = 0$, which shows viscous effect.

STRONG DAMPING

In case of weak damping we neglect the viscous effect, in case of strong damping where $\mu > 0$ we introduce one more term $\mu F(x)$ and the abstract equation will become

$$x + F(x) + \mu F(x) - \frac{1}{\tau}(\alpha + \frac{1}{2}|x'|^2{}_H)\sum_{i=1}^{n}(J(x')(y_i)\delta(y - y_i) - x'') + C_d\, x = p \ldots\ldots (14)$$

And

$$x + x''' + \mu x'''' - \frac{1}{\tau}(\alpha + \frac{1}{2}|x'|^2{}_H)(\sum_{i=1}^{n} \phi x''(y_{i,}t)\delta(y - y_i) - x'') + C_d x = p \ldots\ldots (15)$$

And we conclude that both the equations results are bounded in Hilbert space.

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