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Synchronization technique for fractional order chaotic systems using modified adaptive control method

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ABSTRACT

In the Present chapter authors have investigate the modified Adaptive synchronization method for fractional derivative with unknown parameters. The proposed adaptive modified control method is very convenient and more affective in compression to the present methods for the synchronization of chaotic systems. The numerical simulation results demonstrate that the method is trouble-free to implanted and consistent for synchronization of nonlinear systems.

Keywords: Chaos, Synchronization, Fractional derivative, Adaptive control, modified adaptive control.

1. INTRODUCTION

Nonlinear phenomena are ubiquitous in various areas of scientific fields, and the study of such phenomena has gained huge attention over the last few decades. Nonlinear dynamics is a branch of science and engineering that deals with the study of systems that exhibit nonlinear behaviour and it is an essential tool in understanding the behaviour of chaotic systems. The fractional calculus is a relatively new field of research that deals with the extension of traditional calculus to include fractional orders. Fractional calculus has been successfully used to model various physical phenomena that exhibit memory and long-range interactions, such as anomalous diffusion, viscoelasticity, and fractional quantum mechanics. The incorporation of fractional calculus in nonlinear models has led to the development of more accurate models that can capture the complex behaviour of such systems. One of the most intriguing phenomena in nonlinear dynamics is chaos. Chaos refers to the occurrence of irregular behaviour in a deterministic system i.e. the system's behaviour is entirely determined by its initial conditions.

Many nonlinear systems exhibit chaotic behaviour, including weather patterns, the motion of planets, chemical reactions, and even the behaviour of the human heart. The study of chaos is important in many fields, including physics, mathematics, biology, and engineering, as it provides insight into the behaviour of chaotic systems and can help in the design of more robust and resilient systems Nonlinear models are often used to describe complex systems that exhibit behaviour that cannot be explained by linear models. However, traditional calculus is limited to integer order derivatives and integrals, which may not fully capture the complex behaviour of such systems. By introducing fractional calculus, which allows for the use of non-integer order derivatives and integrals, a more accurate description of nonlinear systems can be obtained. For example, in physics, viscoelastic materials systems have been modelled using fractional calculus. Which exhibit behaviour that cannot be described by traditional linear models. In engineering, fractional order derivative has been used to model complex electrical and mechanical systems, such as the behaviour of fluids in porous media. In biology, fractional calculus has been used to model the dynamics of biological systems, such as the growth of tumours and the spread of diseases. In finance, fractional calculus has been used to model the behaviour of financial markets and to analyze the risk associated with financial instruments. Overall, in nonlinear dynamics with fractional calculus has provided a powerful tool for modelling and analyzing complex systems, and has opened up new avenues for research in many fields. The study of chaotic systems with fractional order has indeed gained much attention in recent years; as such systems demonstrate more complex and unpredictable behaviours than integer order systems. The synchronization of fractional order chaotic systems is an important problem in various applications, ranging from secure communication to control and optimization of complex systems. Constructing a real mathematical model to synchronization the chaotic systems with fractional derivative would require careful consideration of the dynamics and properties of both systems, as well as the coupling mechanism between them. You may need to develop appropriate control algorithms and synchronization schemes to achieve the desired behaviour, and evaluate the performance and robustness of your approach using numerical simulations or experimental data.

In 1990 [1] Pecora and Carroll explained that the synchronization of chaotic systems with different initial conditions is possible through a simple control method. It has attracted a huge compact of interest surrounded by researchers from different fields due to its huge applications in physical & chemical system [2-3], ecological system [4], secure communication, brain activity modelling and system identification ([5-6]) etc. In recent years a variety of synchronization method such as time delay feedback approach [7-8], linear and non linear feedback synchronization [9-11], active and adaptive control [12-16], sliding mode and back stepping control [17-20], projective synchronization and phase [21-24] etc, and recently active control technique has been applied to synchronize between different systems. This method is mostly used on the stability principle of linear differential equation, where the considered Jacobian matrix A of the vector field satisfying $\arg(spec(A)) > \pi\alpha / 2$ [25]. The proposed idea of the study is to examine the time variation of synchronization along with variation of fractional order derivative.

2. PRELIMINARIES:-

2.1 Fractional calculus

Integration and differentiation are generalized in fractional calculus to a non-integer order integro-differential operator ${}_aD_t^p$, which is defined by

$$ {}_aD_t^p = \begin{cases} \dfrac{d^p}{dt^p}, & R(p) > 0, \\ 1, & R(p) = 0, \\ \displaystyle\int_a^t (d\tau)^{-p} & R(p) < 0, \end{cases} \tag{1} $$

where p is the fractional order derivative, the real part of p is denoted by $R(p)$ and t is the moving upper terminal & $a < t$, a is the lower terminal.

Riemann-Liouville fractional derivative definition defined by

$$ {}_aD_t^p x(t) = \frac{d^n}{dt^n} j_t^{n-p} x(t), \quad p > 0, \tag{2} $$

$n = \lceil p \rceil$ i.e., n is the first integer which is not less than p, Riemann-Liouville integral operator j_t^{α} with α order defined as follows

$$ j_t^{\alpha}\varphi(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{\varphi(\tau)}{(t-\tau)^{1-\alpha}} d\tau, \ 0 < \alpha < 1, \tag{3} $$

The Caputo derivative defined as"

$$ {}^cD_t^p x(t) = j_t^{n-p} x^{(n)}(t), \quad p > 0, $$

Lemma 1: If $q > p \geq 0$, r and t are integers where $0 \leq r-1 \leq q < r$, $0 \leq t-1 \leq p < t$, then we obtain [26]

$$ {}_aD_t^q \left\{ {}_aD_t^{-p}\left(f(t)\right) \right\} = {}_aD_t^{q-p}\left(f(t)\right) \tag{4} $$

Lemma 2: If $p, q \geq 0$, r and t are integers such that $0 \leq r-1 \leq q < r$, $0 \leq t-1 \leq p < t$, then we obtain [26]

$$ {}_aD_t^q \left\{ {}_aD_t^{p}\left(f(t)\right) \right\} = {}_aD_t^{p+q}\left(f(t)\right) - \sum_{j=1}^{n} \left[{}_aD_t^{p-j}\left(f(t)\right) \right]_{t=a} \frac{(t-a)^{-q-j}}{\Gamma(1-q-j)} \tag{5} $$

2.2 Problem description

Let us consider the derive and the response system respectively as follows:

$$ D_t^p m = u(m) + U(m)\alpha \tag{6} $$

$$ D_t^p s = v(s) + V(s)\beta + Nc, \tag{7} $$

Here the state vectors $m, s \in R^n$, the unknown parameters α and $\beta \in R^m$ ar, $u(m)$ and $v(s)$ are $n \times 1$ matrices, $U(m)$ and $V(s)$ are the $n \times m$ matrices, the elements $U_{ij}(m)$ in matrix $U(m)$ and $V_{ij}(s)$ in matrix $V(s)$ satisfy $U_{ij}(m) \in L_\infty, \forall m \in R^n$ and $V_{ij}(s) \in L_\infty, \forall s \in R^n$ respectively.

Let, $\varepsilon = y - x$ is the synchronization error vector. Our aim is to construct a controller Nc such that the response system's trajectory (7) approaches asymptotically to drive system (6) i.e., $\lim_{t\to\infty}\|\varepsilon\| = \lim_{t\to\infty}\|s(t, s_0) - m(t, m_0)\| = 0$.

2.3 Modified Adaptive synchronization controller design for Synchronization

Theorem 1: If nonlinear controllers are taken in system (7) as

$$Nc = u(m) + U(m)\alpha - v(s) - V(s)\beta + D_t^{p-1}\left[U(m)\varepsilon_\alpha - V(s)\varepsilon_\beta - (D_t^{p-1}\varepsilon(t))\frac{(t)^{-(p-1)-1}}{\Gamma(-(p-1))} - \varepsilon k\right] \tag{8}$$

and parameter-adaptive laws are considerd as

$\dot{\overline{\alpha}} = -([U(m)]^T\varepsilon + \varepsilon_\alpha)$, $\dot{\overline{\beta}} = ([V(s)]^T\varepsilon - s_\beta)$ (9) Where

the parameters constant $\varepsilon_\alpha = (\overline{\alpha} - \alpha)$, and $\varepsilon_\beta = (\overline{\beta} - \beta)$

then the response system (7) can globally and asymptotically synchronize the derive system (6), satisfying $\lim_{t\to\infty}(\overline{\alpha} - \alpha) = \lim_{t\to\infty}(\overline{\beta} - \beta) = 0$,

where $k > 0$ is a constant and $\overline{\alpha}$, $\overline{\beta}$ are the estimated parameters of α and β."

Proof: By equations (6), (7) & (8) we get

$$D_t^p\varepsilon(t) = D_t^{p-1}\left[U(m)\varepsilon_\alpha - V(s)\varepsilon_\beta - (D_t^{p-1}\varepsilon(t))\frac{(t)^{-(p-1)-1}}{\Gamma(-(p-1))} - \varepsilon k\right], \tag{10}$$

let us consider the Lyapunov function as

$$L(\varepsilon, \varepsilon_\alpha, \varepsilon_\beta) = \frac{1}{2}\left[\varepsilon^T\varepsilon + \varepsilon_\alpha{}^T\varepsilon_\alpha + \varepsilon_\beta{}^T\varepsilon_\beta\right] = \frac{1}{2}\left[\varepsilon^T\varepsilon + (\overline{\alpha} - \alpha)^T(\overline{\alpha} - \alpha) + (\overline{\beta} - \beta)^T(\overline{\beta} - \beta)\right], \tag{11}$$

where $Lp(\varepsilon, \varepsilon_\alpha, \varepsilon_\beta) \in R^n$, and the time derivative of $Lp(\varepsilon, \varepsilon_\alpha, \varepsilon_\beta)$ along the trajectory of the error dynamics system (11) is as fallows

$$\dot{L}(\varepsilon, \varepsilon_\alpha, \varepsilon_\beta) = [e^T\varepsilon + (\overline{\alpha} - \alpha)^T\dot{\overline{\alpha}} + (\overline{\beta} - \beta)^T\dot{\overline{\beta}}]. \tag{12}$$

Using Lemma2 in equation (12) we get

$$\dot{L}(\varepsilon, \varepsilon_\alpha, \varepsilon_\beta) = \left(\left[D_t^{1-p}\left(D_t^p\varepsilon\right) + \left(D_t^{p-1}\varepsilon(t)\right)\frac{(t)^{-(p-1)-1}}{\Gamma(-(p-1))}\right]^T\varepsilon + (\overline{\alpha} - \alpha)^T\dot{\overline{\alpha}} + (\overline{\beta} - \beta)^T\dot{\overline{\beta}}\right) \tag{13}$$

From equations (9) and (12), we get

$$\begin{aligned}\dot{L}(\varepsilon, \varepsilon_\alpha, \varepsilon_\beta) = &\left[D_t^{1-p}\left(D_t^{p-1}\left[U(m)\varepsilon_\alpha - V(s)\varepsilon_\beta - (D_t^{p-1}\varepsilon(t))\frac{(t)^{-(p-1)-1}}{\Gamma(-(p-1))} - \varepsilon k\right]\right) + \left(D_t^{p-1}\varepsilon(t)\right)\frac{(t)^{-(p-1)-1}}{\Gamma(-(p-1))}\right]^T\varepsilon \\ &+ (\overline{\alpha} - \alpha)^T\dot{\overline{\alpha}} + (\overline{\beta} - \beta)^T\dot{\overline{\beta}}\end{aligned} \tag{14}$$

Since $\forall p \in [0,1]$, $(1 - p) > 0$ and $(p - 1) < 0$. from Lemma1 and equation (9), the equation (14) reduces to

$$\begin{aligned}\dot{L}(\varepsilon, \varepsilon_\alpha, \varepsilon_\beta) = &\left[\left(U(m)\varepsilon_\alpha - V(s)\varepsilon_\beta - (D_t^{p-1}\varepsilon(t))\frac{(t)^{-(p-1)-1}}{\Gamma(-(p-1))} - \varepsilon k\right) + \left(D_t^{p-1}\varepsilon(t)\right)\frac{(t)^{-(p-1)-1}}{\Gamma(-(p-1))}\right]^T\varepsilon \\ &- (\overline{\alpha} - \alpha)^T([U(m)]^T\varepsilon + \varepsilon_\alpha) + (\overline{\beta} - \beta)^T([V(s)]^T\varepsilon - \varepsilon_\beta)\end{aligned}$$

$$=[\varepsilon_\alpha{}^T U(m)^T - \varepsilon_\beta{}^T V(s)^T - k\varepsilon^T]\varepsilon - \varepsilon_\alpha{}^T [U(m)]^T \varepsilon - \varepsilon_\alpha{}^T \varepsilon_\alpha + \varepsilon_\beta{}^T [V(s)]^T \varepsilon - \varepsilon_\beta{}^T \varepsilon_\beta$$
$$= -k\varepsilon^T \varepsilon - \varepsilon_\alpha{}^T \varepsilon_\alpha - \varepsilon_\beta{}^T \varepsilon \le 0 \ . \tag{15}$$

Since $\dot{L} \in R^n$ is negative definite function and $L \in R^n$ is the positive definite function, so as per Lyapunov stability theorem [27-28] system (7) is synchronized to system (6) globally and asymptotically. We can also see that the error systems ε and parameters' estimation errors $\varepsilon_\alpha, \varepsilon_\beta$ tends to zero as per time increases.

3. SYSTEMS DESCRIPTIONS

3.1 Fractional order chaotic system

A three-dimensional autonomous system that introduces the necessary nonlinearity for folding trajectories using one quadratic term and two multipliers was studied by Lui et al. in 2009 [29–30]. The offered chaotic system (24) is a new chaotic system with a chaotic attractor which is similar to the Lorenz's chaotic attractors. The following fractional order differential equation in nonlinear form describes this.

$$\begin{cases} \dfrac{d^{p_1} m_1}{dt^{p_1}} = (-\alpha_1 m_1 - \alpha_4 m_2{}^2) \\ \dfrac{d^{p_2} m_2}{dt^{p_2}} = (\alpha_2 m_2 + \alpha_5 m_1 m_3) \\ \dfrac{d^{p_3} m_3}{dt^{p_3}} = (-\alpha_3 m_3 - \alpha_6 m_1 m_2) \end{cases} \tag{16}$$

The chaotic attractor of the system (16) is described by the figure1 with the parameters $\alpha_1 = 1, \alpha_2 = 2.5, \alpha_3 = 5, \alpha_4 = 1, \alpha_5 = 4, \alpha_6 = 2$, and (0.2, 0 0.5) are the initial values of state vectors m_1, m_2, m_3 .

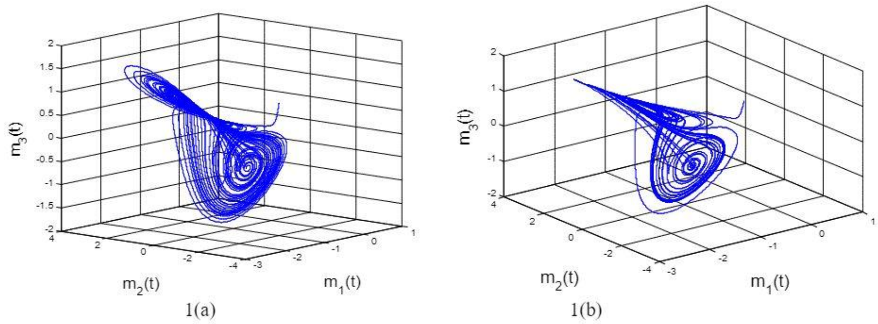


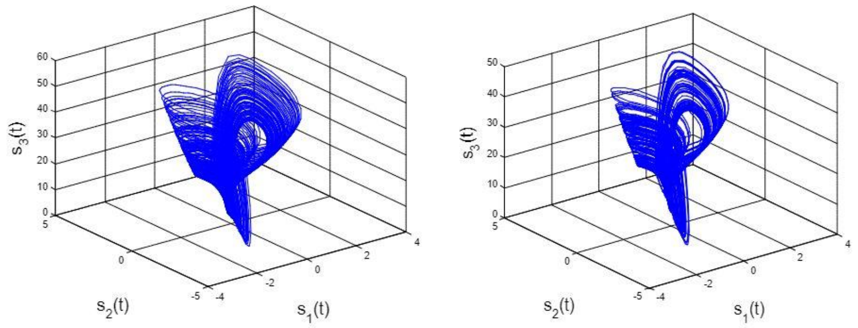
Figure 1. Chaotic attractor of the system (16):**1(a)** for $p_i = 1$ & **1(b)** for $p_i = 0.98$, $\forall i = 1,2,3$

3.2 Fractional order novel chaotic system

The expression of novel chaotic system [31-32] with fractional order is

$$\begin{cases} \dfrac{d^{p_1} s_1}{dt^{p_1}} = \beta_1 (s_2 - s_1) \\ \dfrac{d^{p_2} s_2}{dt^{p_2}} = (\beta_2 s_1 - \beta_3 s_1 s_3) \\ \dfrac{d^{p_3} s_3}{dt^{p_3}} = ((\exp)^{s_1 s_2} - \beta_4 s_3) \end{cases} \tag{17}$$

where s_1, s_2, s_3 are the state variables and $\beta_1, \beta_2, \beta_3, \beta_4$ are the parameters.
System (17) produces the chaotic attractor according to the parameters. $\beta_1 = 10, \beta_2 = 40, \beta_3 = 2, \beta_3 = 2.5$ with the initial conditions of state space (2.2, 2.4, 28).



1(a) 1(b)

Figure 2. Chaotic attractor of the system (17): **2(a)** for $p_i = 1$ & **2(b)** for $p_i = 0.98$, $\forall i = 1,2,3$

4. Synchronization between different fractional order autonomous chaotic systems by applying the modified adaptive control technique

This section examines how the innovative fractional order 3D autonomous chaotic systems (16) and (17) synchronize. It is assumed that system (16) drive the chaotic system (17). Now the system (16) and system (17) are defined with control parameters $Nc = [Nc_1, Nc_2, Nc_3]^T$ as

$$\begin{cases} \dfrac{d^{p_1} m_1}{dt^{p_1}} = (-\alpha_1 m_1 - \alpha_4 m_2^{\ 2}) \\ \dfrac{d^{p_2} m_2}{dt^{p_2}} = (\alpha_2 m_2 + \alpha_5 m_1 m_3) \\ \dfrac{d^{p_3} m_3}{dt^{p_3}} = (-\alpha_3 m_3 - \alpha_6 m_1 m_2) \end{cases} \tag{18}$$

and

$$\begin{cases} \dfrac{d^{p_1} s_1}{dt^{p_1}} = \beta_1 (s_2 - s_1) + Nc_1 \\ \dfrac{d^{p_2} s_2}{dt^{p_2}} = (\beta_2 s_1 - \beta_3 s_1 s_3) + Nc_2 \\ \dfrac{d^{p_3} s_3}{dt^{p_3}} = ((\exp)^{s_1 s_2} - \beta_4 s_3) + +Nc_3 \end{cases} \tag{19}$$

The error function can be described as

$$\begin{cases} \varepsilon_1 = s_2 - m_1 \\ \varepsilon_2 = s_2 - m_2 \\ \varepsilon_3 = s_3 - m_3 \end{cases} \tag{20}$$

So according to the **theorem 1,** the controller function are taken as

$$
\begin{aligned}
Nc_1 &= (-\alpha_1 m_1 - \alpha_4 m_2^{\,2}) - \beta_2(s_2 - s_1) \\
&\quad + D^{p_1-1}\left[-\varepsilon_{\alpha_1} m_1 - \varepsilon_{\alpha_4} m_2^{\,2} - \varepsilon_{\beta_1} s_2 - (D_t^{p_1-1}\varepsilon_1(t))\frac{(t)^{-(p_1-1)-1}}{\Gamma(-(p_1-1))} - \varepsilon_1 k_1 \right]
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
Nc_2 &= (\alpha_2 m_2 + \alpha_5 m_1 m_3) - (\beta_2 s_1 - \beta_3 s_1 s_3) \\
&\quad + D^{p_2-1}\left[\varepsilon_{\alpha_2} s_2 + \varepsilon_{\alpha_6} m_1 m_3 - \varepsilon_{\beta_2} s_1 + \varepsilon_{\beta_3} s_1 s_3 - (D_t^{p_2-1}\varepsilon_2(t))\frac{(t)^{-(p_2-1)-1}}{\Gamma(-(p_2-1))} - \varepsilon_2 k_2 \right]
\end{aligned} \tag{22}
$$

$$
\begin{aligned}
Nc_3 &= (-\alpha_3 m_3 - \alpha_6 m_1 m_2) - (\exp^{s_1 s_2} - \beta_4 s_3) \\
&\quad + D^{p_3-1}\left[-\varepsilon_{\alpha_3} m_3 - \varepsilon_{\alpha_6} m_1 m_2 + \varepsilon_{\alpha_4} s_3 - (D_t^{p_3-1}\varepsilon_3(t))\frac{(t)^{-(p_3-1)-1}}{\Gamma(-(p_3-1))} - \varepsilon_3 k_3 \right]
\end{aligned} \tag{23}
$$

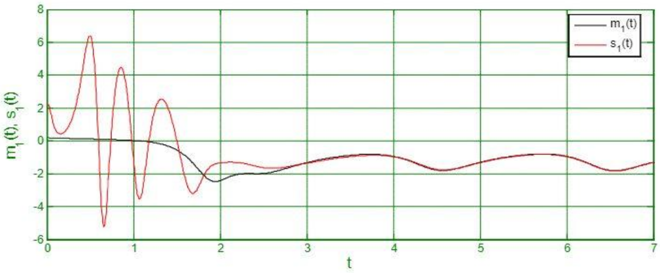
where k_1, k_2, k_3 are the real positive constants, and $\varepsilon_{\alpha_1} = (\overline{\alpha}_1 - \alpha_1)$, $\varepsilon_{\alpha_2} = (\overline{\alpha}_2 - \alpha_2)$, $\varepsilon_{\alpha_3} = (\overline{\alpha}_3 - \alpha_3)$, $\varepsilon_{\alpha_4} = (\overline{\alpha}_4 - \alpha_4)$, $\varepsilon_{\alpha_5} = (\overline{\alpha}_5 - \alpha_5)$, $\varepsilon_{\alpha_6} = (\overline{\alpha}_6 - \alpha_6)$, $\varepsilon_{\beta_1} = (\overline{\beta}_1 - \beta_1)$, $\varepsilon_{\beta_2} = (\overline{\beta}_2 - \beta_2)$, $\varepsilon_{\beta_3} = (\overline{\beta}_3 - \beta_3)$, $\varepsilon_{\beta_4} = (\overline{\beta}_4 - \beta_4)$

The estimated parameters are calculated as

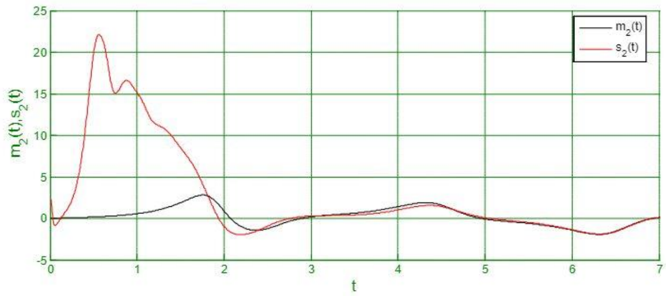
$$
\begin{cases}
\overline{\alpha}_1 = (m_1\varepsilon_1 - \varepsilon_{\alpha_1}), \overline{\alpha}_2 = (-m_2\varepsilon_2 - \varepsilon_{\alpha_2}), \overline{\alpha}_3(m_3\varepsilon_3 - \varepsilon_{\alpha_3}), \\
\overline{\alpha}_4 = (m_2^{\,2}\varepsilon_1 - \varepsilon_{\alpha_4}), \overline{\alpha}_5 = (-m_1 m_3\varepsilon_2 - \varepsilon_{\alpha_5}), \overline{\alpha}_6 = (m_1 m_2\varepsilon_3 - \varepsilon_{\alpha_6}) \\
\overline{\beta}_1 = ((s_2 - s_1)\varepsilon_1 - \varepsilon_{\beta_1}), \overline{\beta}_2 = (s_1\varepsilon_2 - \varepsilon_{\beta_2}), \overline{\beta}_3 = (-s_1 s_3\varepsilon_2 - \varepsilon_{\beta_3}), \overline{\beta}_4 = (-s_3\varepsilon_3 - \varepsilon_{\beta_4}),
\end{cases} \tag{24}
$$

4.1 Numerical Simulation and results

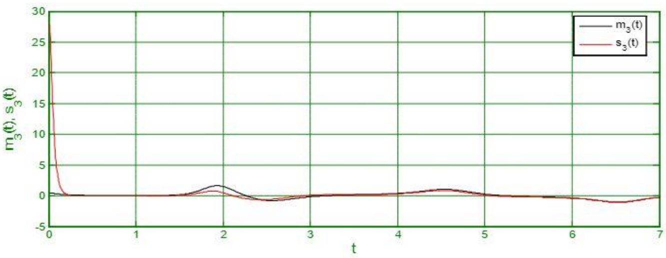
In numerical simulations for the adaptive synchronization of drive and response systems, the unknown parameter vectors of the drive and response systems are taken same as the section 3 and the initial values of the estimated unknown parameter vectors of drive and response system are taken as $(\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0, \alpha_5 = 0, \alpha_6 = 0)$ and $(\beta_1 = 0, \beta_2 = 0, \beta_3 = 0, \beta_4 = 0)$ respectively. The initial values of drive system (24) and response system (25) of the state vectors are also taken similar as to the section 3 and the initial value of error vector and constant k are $\varepsilon = [-2, -2.4, -27.5]^T$ and $k = [1,1,1]^T$ respectively. The response system (24) is synchronized with the drive system (25) as shown in Fig.3 to Fig.5 for the order of derivatives $p = [1.0, 1.0, 1.0]^T$, $p = [0.98, 0.98, 0.98]^T$ and $p = [0.95, 0.95, 0.95]^T$ respectively, and it is seen that the synchronization error vectors converge asymptotically to zero. Fig.3(e) & 3(f), Fig.4(e) & 4(f) and Fig.5(e) & 5(f) show the variations of estimated parameter vectors which are converge to the original parameter vectors for both the drive and response systems at the order of derivatives $p = [1.0, 1.0, 1.0]^T$, $p = [0.98, 0.98, 0.98]^T$ and $p = [0.95, 0.95, 0.95]^T$ respectively.



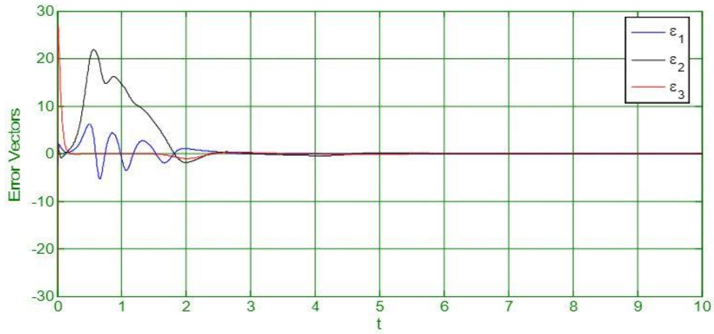
3(a) State trajectory between state vectors m_1 and s_1



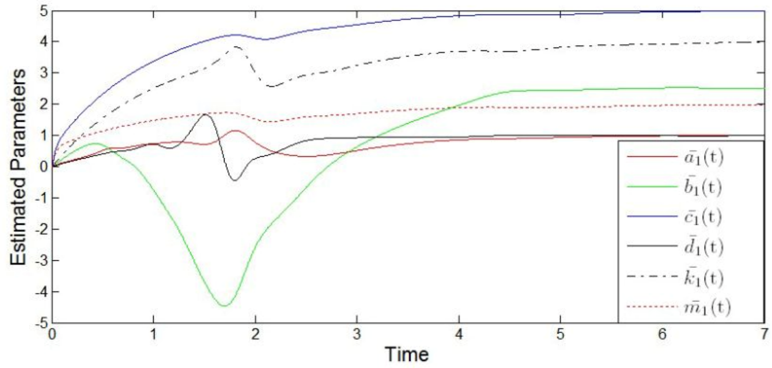
3(b) State trajectory between state vectors m_2 and s_2



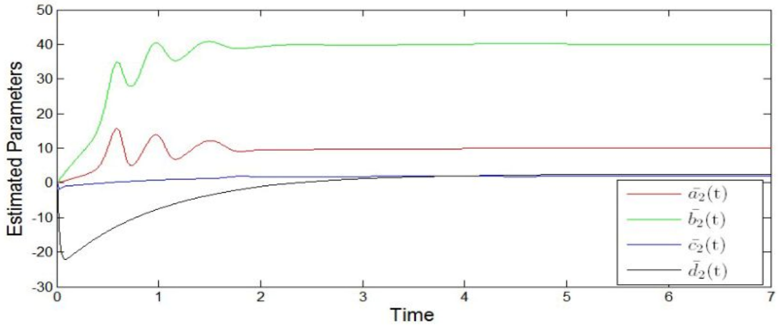
3(c) State trajectory between state vectors m_3 and s_3



3(d) State trajectory between error vectors $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε_4

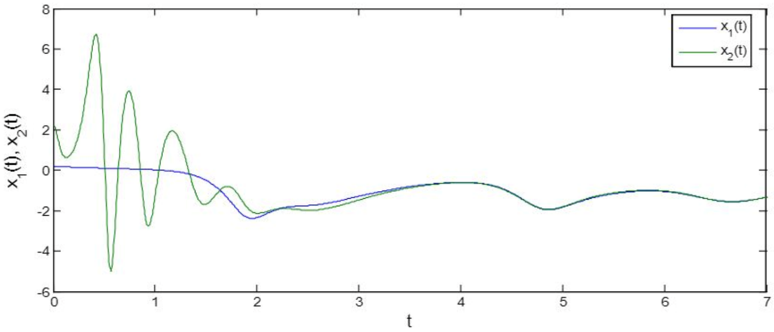


3(e) State trajectories of adaptive parameters $\overline{a}_1, \overline{b}_1, \overline{c}_1, \overline{d}_1, \overline{k}_1$ and $\overline{m}_1$

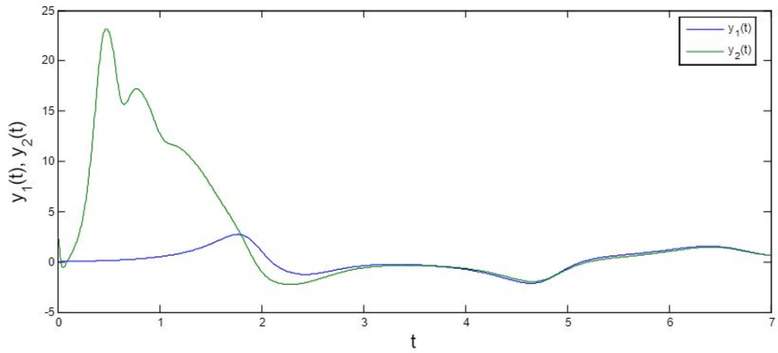


3(f) State trajectories of adaptive parameters $\overline{a}_2, \overline{b}_2, \overline{c}_2$, and $\overline{d}_2$

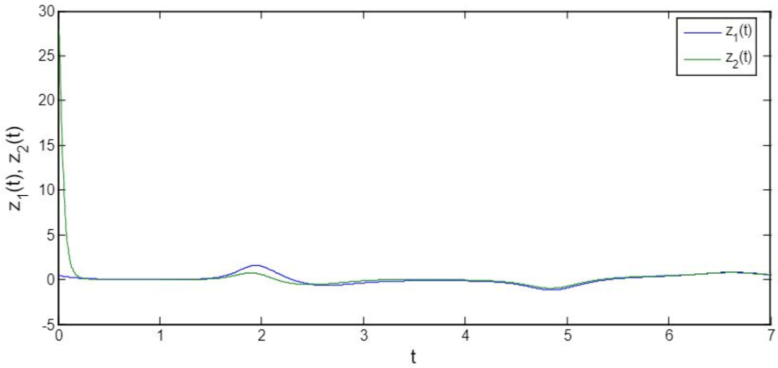
FIGURE 3 State trajectories of the state vectors, error system and estimated parameters of drive system (18) and response system (19) for $p = [1.0, 1.0, 1.0]^T$.



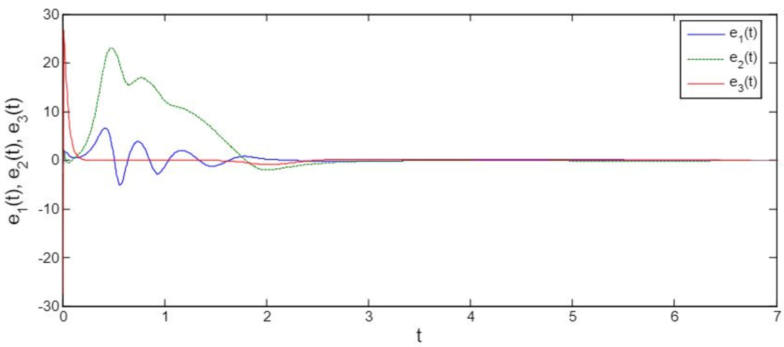
4(a) State trajectory between state vectors x_1 and x_1



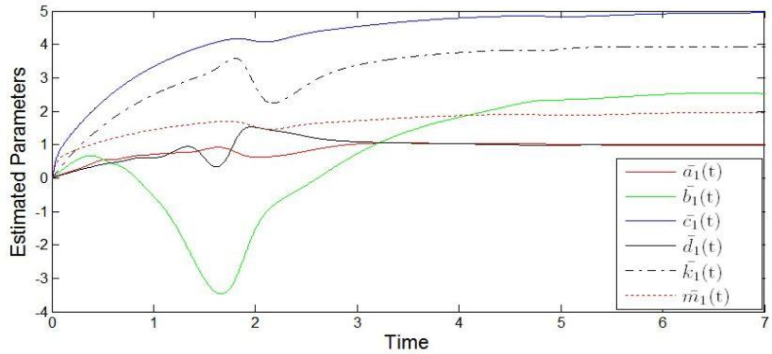
4(b) State trajectory between state vectors y_1 and y_2



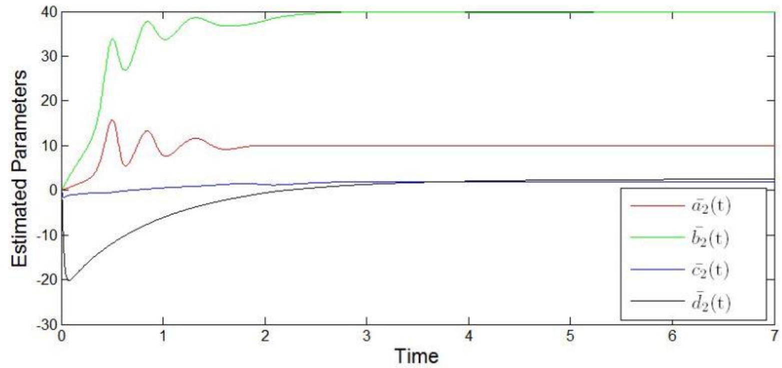
4(c) State trajectory between state vectors z_1 and z_2



4(d) State trajectory between error vectors e_1, e_2, e_3 and e_4

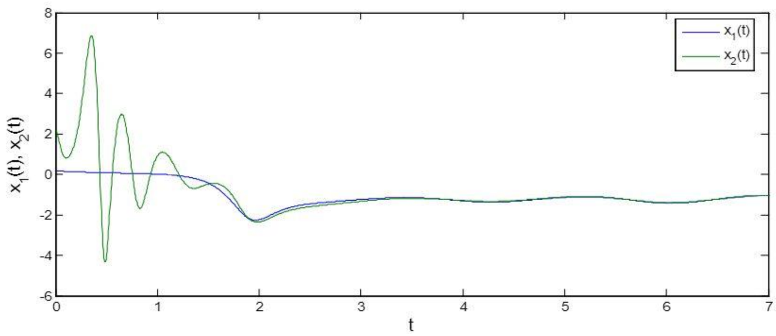


4(e) State trajectories of adaptive parameters $\overline{a}_1, \overline{b}_1, \overline{c}_1, \overline{d}_1, \overline{k}_1$ and $\overline{m}_1$

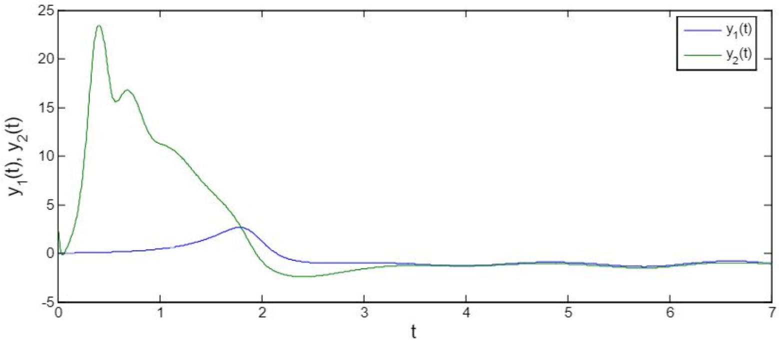


4(f) State trajectories of adaptive parameters $\overline{a}_2, \overline{b}_2, \overline{c}_2$, and $\overline{d}_2$

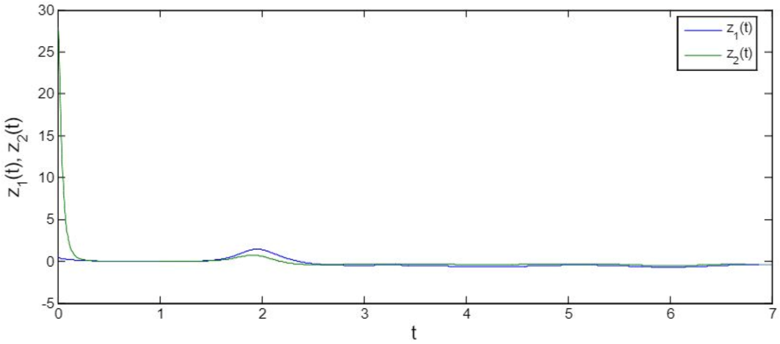
Fig. 4 State trajectories of the state vectors, error system and estimated parameters of drive system (18) and response system (19) for $p = [0.98, 0.98, 0.98]^T$.



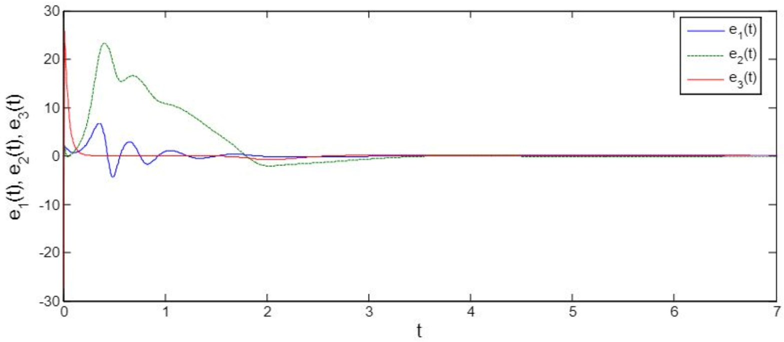
5(a) State trajectory between state vectors x_1 and x_1



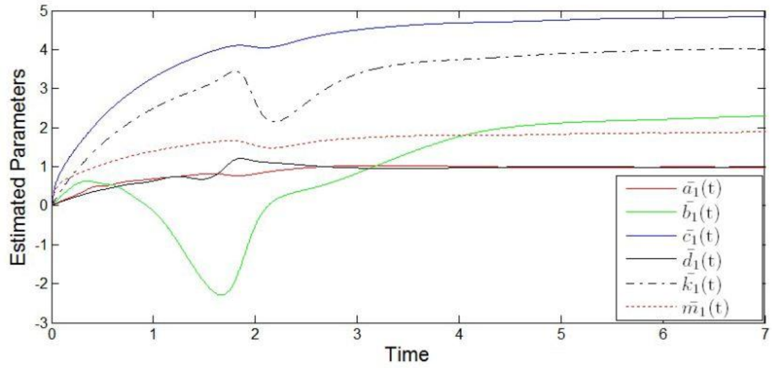
5(b) State trajectory between state vectors y_1 and y_2



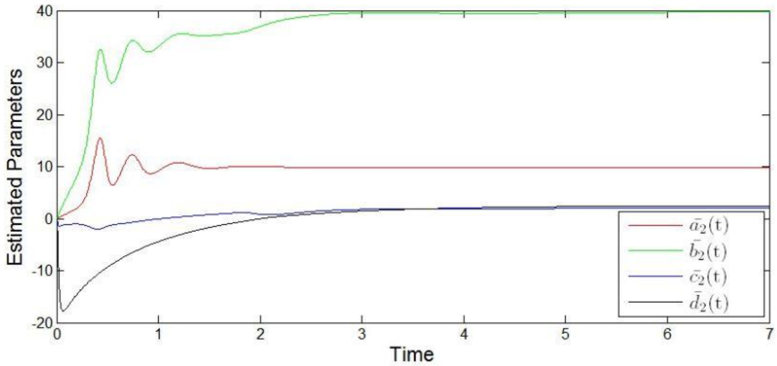
5(c) State trajectory between state vectors z_1 and z_2



5(d) State trajectory between error vectors e_1, e_2, e_3 and e_4



5(e) State trajectories of adaptive parameters $\overline{a}_1, \overline{b}_1, \overline{c}_1, \overline{d}_1, \overline{k}_1$ and $\overline{m}_1$



5(f) State trajectories of adaptive parameters $\overline{a}_2, \overline{b}_2, \overline{c}_2,$ and $\overline{d}_2$

FIGURE 5. State trajectories of the state vectors, error system and estimated parameters of drive system (18) and response system (19) for $p = [0.95, 0.95, 0.95]^T$.

5. CONCLUSION

The present investigation has attained accomplishment in two significant capacities. Firstly it is successfully carried out the study of a modified adaptive synchronization method is proposed for applying in fractional order chaotic systems. The Adaptive controller and parameters update law are designed properly to synchronize/anti synchronizes two different pair of chaotic systems based on the Lyapunov direct stability theory. The proposed modified method evidently exhibits that its simplicity and suitability for a large class of integer as well as fractional order chaotic systems.

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