ISSN: 2178-7727

Shear Deformation under Bending Conditions Analysis of Cantilever FGM Beams

Aissa BOUSSOUAR, Bachir TAALLAH, Abdelhak KHECHAI, Ali ZAIDI.

Civil Engineering Research Laboratory, Mohamed Khider University of Biskra, Biskra, Algeria.
E-mail: batiaissa85@gmail.com, b.taallah@gmail.com, abdelhak_khechai@hotmail.fr
Structures Rehabilitation and Materials Laboratory (SREML), University of Laghouat, Laghouat, Algeria.
E-mail: a.zaidi@lagh-univ.dz

ABSTRACT

In this paper, a static analysis to establish a mathematical model using high order bending theories considering shear strains in displacement fields which have not been taken into account by other theories for functionally graded material (FGM) cantilever beams under bending. A new polynomial shear function satisfied the stress-free boundary conditions is used in this investigation. These theories do not require a shear correction factor and consider a hyperbolic shape function. Material properties are assumed to vary in the thickness direction, a simple power-law distribution in terms of volume fractions of constituents is considered. Two illustrative cases, a cantilever FGM beam subjected to a concentrated shear force at the free end, and also as a cantilever FGM beam with uniformly distributed load are presented the originality of this research work, in this investigation. The mathematical model is established by differential equations which are derived by the principle of virtual work. Equilibrium equations and boundary conditions are introduced. The solution model is based on a variation approach (integrals) to predict the field component of displacements and the basic constitutive laws. The solution of the analytical model is presented. The results in terms of displacement fields including rotation of the section, and shear stresses, predicted from the proposed model, are presented.

Keywords: High order theories and bending. Mathematical model. Cantilever Beams. New polynomial shear function. Displacement fields. Shear stress.

INTRODUCTION

Reddy (Reddy et al., 2000) presented theoretical formulation and element models finite based on third order theory to analyze FGM plates. Ghugal et al. (Ghugal et al., 2009; Dahake et al., 2012) proposed elasticity analytical solutions for the static flexure analysis of thick isotropic beams subjected to concentrate load and/or uniform distribution load with various end conditions and compared the displacement field of illustrative cases to those of Timoshenko and solutions in literature.

Gandhe et al (Gandhe et al., 2018) studied static flexural analysis of thick beams using hyperbolic shear deformation theory. The virtual work principle was used to derive governing equations of beams. Displacements and stresses obtained for beams under variable loads were compared to those predicted from Timoshenko and higher order theories.

Guenfoud et al (Guenfoud et al., 2016) proposed a new polynomial shear function which satisfies the stress-free boundary conditions; however, this theory has strong similarities with Timoshenko beam theory in some concepts such as equations of movement, boundary conditions and stress resultant expressions.

Aydogdu et al (Aydogdu et al., 2007) have studied the free vibration analysis of functionally graded beams, governing equations by Hamilton's principle and Navier type solution to obtain frequencies of FG beam with simply supported ends. As mainly results, the classical beam theory (CBT) gives higher results; furthermore, the difference between the CBT and higher order theories is increasing with increasing the mode number.

Y. Huang et al. (Huang et al., 2010) were studied the free vibration of axially FGMs with non-uniform cross-sections. Solve free vibration of Euler–Bernoulli beams with continuously varying flexural rigidity and mass density, by using the integration technique to transform the differential governing equations into the Fredholm integral equations, for obtain natural frequencies.

ISSN: 2178-7727

Benatta et al. (Benatta et al., 2009; Benatta et al., 2008) carried out a static analysis of short FGM beams with simply supported ends including warping and shear deformations effects. The results obtained showed that the proposed formulation permits, for warping of the cross-section of the FGM beam, to eliminate the necessity to use the arbitrary shear correction coefficients. Also, studied the response of the bending of short hybrid composite beams varying fibers spacing. Authors proved that the hybrid FGM with a fiber volume fraction variation, can improve the beam design.

Bouremana et al. (Bouremana et al., 2013) were presented a new first shear deformation theory based on neutral surface position for FGM beams, in which a system of equations of motion equations are derived by Hamilton's principle, several comparison studies are investigated, the effects of different parameters of the beam on the bending and free vibration responses.

Akbas (Akbas et al., 2018) was analyze large deflections of a fiber reinforced composite cantilever beam under point loads. The problem is solved by FEM and considering the total Lagrangian approach with Newton-Raphson iteration method. Simsek (Şimşek et al., 2009) was studied functionally graded (FG) simply-supported beam using Ritz method,

under bending. The solution carried out research works on the displacements and the stresses.

Zaoui et al. (Ziou et al., 2021) were carried out research works on the bending and free vibration of functionally graded material (FGM) beams. Equations of motion are derived from Hamilton’s principle and Navier-type analytical solutions.

Zhong et al. (Zhong et al., 2007) proposed elasticity analytical solutions for functionally graded cantilever beams under different load types.

Ziou et al (Zaoui et al., 2017) analyzed a cantilever functionally graded material (FGM) beam subjected to a concentrated force at the free end for different ratio of length-to-thickness chosen using finite element method. The governing equations and boundary conditions are derived from virtual work principle. The numerical results prove that deflections are more highly for metal rich beam compared to those of ceramic rich beam, furthermore, this deflection increases as the power law index increases. In addition, the Timoshenko beam theory is used to examine deflections, stresses distribution, and strains on the beam.

However, these previous research works as seen in this literature review did not investigate sufficiently the cantilever beams taken into account the shear deformations in the displacement fields.

The aim of this paper is to propose mathematical model for cantilever FGM beams under bending, considering shear strains in the displacement fields is efficient to predict the solution. Briefly, the manuscript is organized by description of material properties as well as the proposed mathematical model for two cases; Case1 a cantilever beam with concentrated load (P) at free end. In addition; Case2 a cantilever beam with uniformly distributed load (q). Based on high-order bending theories to determine the governing differential equations which are derived by the virtual work principle and solved by integrals. Then, a numerical and discussion of results, in terms of displacement fields, shear stresses. This proposed analytical model using a new polynomial shear function is a new contribution.

MATERIAL AND CROSS SECTION PROPERTIES

Material Properties

The stiffness coefficients E(z) and G(z) obtained based on the mixing rule of constituents, can be written as follows:

$$E(z) = \mathrm{E_F V_F(z)} + \mathrm{E_M}(1 - \mathrm{V_F(z)}) \tag{1}$$

$$\frac{1}{G(z)} = \frac{\mathrm{V_F(z)}}{\mathrm{G_F}} + \frac{1-\mathrm{V_F(z)}}{\mathrm{G_M}} \tag{2}$$

Where V_F: Fiber volume fraction; E_F: Young modulus of fiber; E_M: Young modulus of matrix; G_F: Fiber shear modulus; G_M: Matrix shear modulus.

Equation (3) which presents a simple power law with index *n* on the z-axis and the height of the section, following the constituent mixing rule, can be written as:

$$\mathrm{V_F(z)} = \mathrm{V_2} + (\mathrm{V_1} - \mathrm{V_2})\left(2\frac{\mathrm{z}}{\mathrm{h}}\right)^{\mathrm{n}} \tag{3}$$

The fiber volume fraction V_F varies through the thickness, with values $V_1=V_F(h/2)$ and $V_2=V_F(0)$.

ACTA SCIENTIAE

ISSN: 2178-7727

Cross Section Properties

Equation (4) illustrates the stiffness coefficients of beam A_{11}, B_{11} and D_{11} which are extension, bending-extension coupling, and bending stiffness coefficients, respectively; its expression under integrals between the half–heights of the section is given by:

$$\{A_{11}, B_{11}, D_{11}\} = b\int_{z=\frac{-h}{2}}^{z=\frac{h}{2}} \mathrm{E(z)}\{1, \mathrm{z}, \mathrm{z}^2\}\mathrm{dz} \tag{4}$$

Although, $B^a{}_{11}$, $D^a{}_{11}$, $F^a{}_{11}$ which represent the additional coupling coefficients and the bending stiffness depending on the shape function φ(z), are presented under the following integrals:

$$\{B^a{}_{11}, D^a{}_{11}, F^a{}_{11}\} = b\int_{z=\frac{-h}{2}}^{z=\frac{h}{2}} \mathrm{E(z)}\varphi(\mathrm{z})\{1, \mathrm{z}, \varphi(\mathrm{z})\}\,\mathrm{dz} \tag{5}$$

Also, the coefficient of the additional transverse shear stiffness is:

$$A^a{}_{55} = b\int_{z=\frac{-h}{2}}^{z=\frac{h}{2}} \mathrm{G(z)}\left(\frac{\partial\varphi(\mathrm{z})}{\partial z}\right)^2\mathrm{dz} \tag{6}$$

MATHEMATICAL MODEL

Displacements fields

The displacement components are defined from the equations (9) to (11) along the x and z- directions of the beam, when the rotation of the section θ(x) is measured on the mean line:

$$u(\mathrm{x,z}) = \mathrm{u_0(x)} - \mathrm{z}\frac{\partial \mathrm{w(x)}}{\partial x} + \varphi(z)\left[\frac{\partial \mathrm{w(x)}}{\partial x} + \theta(\mathrm{x})\right] \tag{9}$$

$$\mathrm{V(x,z)} = 0 \tag{10}$$

$$\mathrm{w(x,z)} = \mathrm{w_0}(x) \tag{11}$$

While, the normal stress and the shear stress of Hooke’s basic constitutive law are presented as follows:

$$\sigma_{\mathrm{xx}}(x) = \mathrm{E(z)}\varepsilon_{\mathrm{x}} \tag{12}$$

$$\tau_{\mathrm{xz}}(x) = \mathrm{G(z)}\gamma_{\mathrm{xz}} \tag{13}$$

New polynomial shear function

A new polynomial shear function contribution is proposed in this section. This function is given as below:

$$\varphi(z) = \mathrm{z}\left(\frac{13}{12} - \frac{13\mathrm{z}^2}{9h^2}\right) \tag{14}$$

At the both top and bottom fiber where ($z = \pm\frac{\mathrm{h}}{2}$), the first derived of polynomial shear function $\frac{\partial\varphi(z)}{\partial \mathrm{z}} = 0$. In addition, the nullity of the shear stress can be confirmed at theses fibers.

$$\frac{\partial\varphi(z)}{\partial \mathrm{z}} = \frac{13}{12} - \frac{13z^2}{3h^2} \tag{14-a}$$

The second derived for (z=0) involves that the shear stress is maximal at the midline fiber, when φ (z=0) =0.

$$\frac{\partial^2\varphi(z)}{\partial z^2} = -\frac{26\,\mathrm{z}}{3h^2} \tag{14-b}$$

DISPLACEMENTS AND STRAINS

Equilibrium equations

Considering the geometric dimensions of the beam at the boundary restrains, as shown in figure 1 and 2, having a width (b), span length (L), thickness (h), inertia (I), under transverse concentrate load (P) and also a distributed uniform load (q). In this section, it can be noted that only a pure flexure is reported. The virtual work principle applied in brief on the beam is defined as follows:

ISSN: 2178-7727

$$b\int_{x=0}^{x=L}\int_{z=\frac{-h}{2}}^{z=\frac{h}{2}}(\sigma_x\delta\varepsilon_x+\tau_{xz}\delta\gamma_{xz})dxdz-\int_{x=0}^{x=L}q\delta w dx=0 \tag{15}$$

$$b\int_{x=0}^{x=L}(N\frac{\partial\delta u_0}{\partial x}-M_b\frac{\partial^2\delta w_0}{\partial x^2}+M_s\frac{\partial\delta\psi_x}{\partial x}+Q\delta\psi_x)dx-b\int_{x=0}^{x=L}q\delta w_0 dx=0 \tag{15-a}$$

Equations (15-b) and (15-c) present the normal resultant force (N) and the shear resultant force (Q), respectively.

$$N=\int_{\frac{-h}{2}}^{\frac{h}{2}}\sigma_x dz \tag{15-b}$$

$$Q=\int_{\frac{-h}{2}}^{\frac{h}{2}}\frac{\partial\varphi(z)}{\partial z}\tau_{zx}dz \tag{15-c}$$

The bending moment (M_b) and the shear bending moment (M_s), equations (15-d) and (15-e), respectively, are given by:

$$M_b=\int_{\frac{-h}{2}}^{\frac{h}{2}}z\sigma_x dz \tag{15-d}$$

$$M_s=\int_{\frac{-h}{2}}^{\frac{h}{2}}\varphi(z)\sigma_x dz \tag{15-e}$$

Equilibrium equations (16) to (19) are derived by deductions from the integrals:

$$\frac{\partial N(x)}{\partial x}=0 \tag{16}$$

$$\frac{\partial^2 M_b}{\partial x^2}+q=0 \tag{17}$$

$$\frac{\partial^2 M_s}{\partial x^2}+\frac{\partial Q}{\partial x}+q=0 \tag{18}$$

$$\frac{\partial M(x)}{\partial x}-N(x)=0 \tag{19}$$

The equilibrium equation in terms of displacement and shear can be written as below:

$$\frac{\partial^2\varphi_x}{\partial x^2}-\frac{D_{11}}{F_{55}}\left(\varphi_x+\frac{\partial w_0(x)}{\partial x}\right)=0 \tag{20}$$

Also,

$$\frac{\partial^2 w(x)}{\partial x^2}+\frac{\partial\varphi_x}{\partial x}+F_{55}q=0 \tag{21}$$

Where, $D_{11}=\frac{b}{E_x I}$ is the bending stiffness constant of the beam. And , $F_{55}=\frac{1}{hG_{xz}}$ is the shear stiffness constant of the beam.

Boundary conditions

The application of essential boundary conditions (Neumann) and natural (Dirichlet) for displacement and forces with a consideration:

- Case1: Boundary conditions of cantilever beam with concentrated load (P) at free end

$$\frac{\partial w}{\partial x}=\varphi_x=w=0 \quad \text{for } x=L \tag{22}$$

- Case2: Boundary conditions of cantilever beam with uniformly distributed load (q)

$$D_{11}\frac{\partial^3 w(x)}{\partial x^3}=D_{11}\frac{\partial^2\varphi_x}{\partial x^2}=D_{11}\frac{\partial^2 w(x)}{\partial x^2}=D_{11}\frac{\partial\varphi_x}{\partial x}=0 \quad \text{for } x=0 \tag{23}$$

$$\frac{\partial w}{\partial x}=\varphi_x=w=0 \quad \text{for } x=L \tag{24}$$

ISSN: 2178-7727

Illustrative case1:

Equation 25 presents the displacement expression according to the classical beam theory of a cantilever beam with concentrate load P at free end (Figure 1), is given by the followings:

$$\upsilon(\mathrm{x}) = -x(x^2 - 3\mathrm{L}x) \tag{25}$$

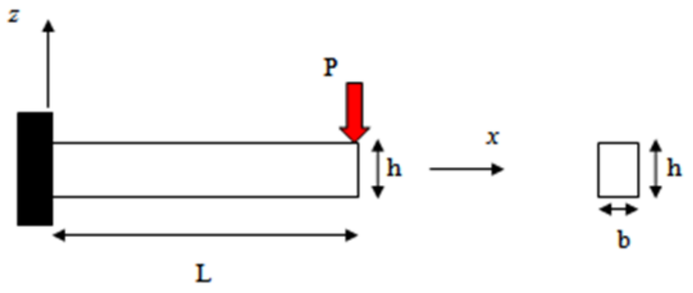

Figure 1 - Cantilever beam with concentrate load P at free end.

Equation 26 and 27 are respectively the dimensionless shear coefficient and the function φ_θ (z) which is derived from the solution, are taken as follows:

$$S_\theta = \frac{3}{4L^2}\frac{{D^a}_{11}{}^2}{{A^a}_{55}D_{11}} \tag{26}$$

$$\varphi_\theta(\mathrm{x}) = x + \frac{1}{\Omega_\theta}(\, cosh\,(\Omega_\theta x) - \; sinh\,(\Omega_\theta x) - 1) \tag{27}$$

Finally, the displacement components are as follows:

$$u(\mathrm{x,z}) = -\mathrm{z}\frac{\mathrm{F_z}L^2}{3bD_{11}}\left(\frac{\partial \upsilon(\mathrm{x})}{\partial x} + 2S_\theta\frac{\partial \varphi_\theta(\mathrm{x})}{\partial x}\right)] + \varphi(\mathrm{z})\frac{\mathrm{F_z}}{\mathrm{b}{D^a}_{11}}\left(\frac{4L^2}{3}\; S_\theta\;\frac{\partial \varphi_\theta(\mathrm{x})}{\partial x}\right)] \tag{28}$$

$$V(\mathrm{x,z}) = 0 \tag{29}$$

$$w(\mathrm{x}) = \frac{\mathrm{F_z}L^2}{3bD_{11}}[\upsilon(\mathrm{x}) + 2S_\theta\,\varphi_\theta(\mathrm{x})] \tag{30}$$

The deformation components along *x* and *z*-directions of the beam are given as follows:

$$\varepsilon(\mathrm{x,z}) = -\mathrm{z}\frac{\mathrm{F_z}L^2}{3bD_{11}}\left(\frac{\partial^2\upsilon(\mathrm{x})}{\partial x^2} + 2S_\theta\frac{\partial^2\varphi_\theta(\mathrm{x})}{\partial x^2}\right) + \varphi(\mathrm{z})\frac{\mathrm{F_z}}{\mathrm{b}{D^a}_{11}}\left(\frac{4L^2}{3}\; S_\theta\;\frac{\partial^2\varphi_\theta(\mathrm{x})}{\partial x^2}\right) \tag{31}$$

In addition, the transverse shear deformation is written as:

$$\gamma^0{}_{\mathrm{xz}}(x,z) = \frac{\partial\,\varphi(z)}{\partial z}\frac{4\mathrm{F_z}L^2}{3\mathrm{b}{D^a}_{11}}S_\theta\frac{\partial\varphi_\theta(\mathrm{x})}{\partial x} \tag{32}$$

Illustrative case2:

Figure2 is a cantilever beam with uniformly distributed load (q), their displacement is derived from the classical beam theory, for the interval ($0 \le x \le \mathrm{L}$), the analytical expression is as follows:

$$\upsilon(\mathrm{x}) = -x^2(x^2 - 4\mathrm{L}x + 6L^3) \tag{33}$$

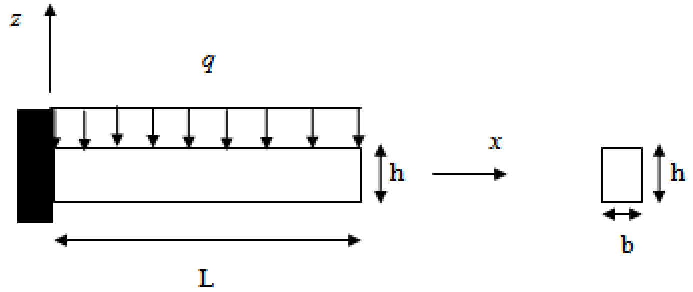

Figure 2 - Cantilever beam with uniformly distributed load q

When taking shear into account, the coefficient without dimensionless can be expressed as the form:

$$S_\theta = \frac{2}{L^2}\frac{{D^a}_{11}{}^2}{{A^a}_{55}D_{11}} \tag{34}$$

ISSN: 2178-7727

The function φ_θ (z) which is derived from the solution can be defined by the following expression:

$$\varphi_\theta(\mathrm{x}) = 1 - (\frac{\mathrm{x}}{\mathrm{L}})^2 + \frac{2\,\Omega_\theta \mathrm{L}\,[\sinh(\Omega_\theta L) - \sinh(\Omega_\theta x)]}{(\mathrm{L}\Omega_\theta)^2 \cosh(L\Omega_\theta)} + \frac{2\cosh\Omega_\theta (L-x)}{(\mathrm{L}\Omega_\theta)^2\cosh(L\Omega_\theta)} \quad (35)$$

The analytical solution of the displacement components, are written as illustrated by equations (36) to (38):

$$u(\mathrm{x},\mathrm{z}) = -\mathrm{z}\frac{\mathrm{F_z}L^2}{8bD_{11}}\left(\frac{\partial \upsilon(\mathrm{x})}{\partial x} + 2S_\theta\frac{\partial \varphi_\theta(\mathrm{x})}{\partial x}\right)] + \varphi(\mathrm{z})\frac{(\mathrm{F_z}=\mathrm{qL})}{\mathrm{b}D^a{}_{11}}\left(\frac{L^2}{2}\,S_\theta\,\frac{\partial \varphi_\theta(\mathrm{x})}{\partial x}\right)] \quad (36)$$

$$V(\mathrm{x},\mathrm{z}) = 0 \quad (37)$$

$$w(\mathrm{x}) = \frac{\mathrm{F_z}L^2}{8bD_{11}}[\upsilon(\mathrm{x}) + 2S_\theta\,\varphi_\theta(\mathrm{x})] \quad (38)$$

However, the deformation components along *x* and *z*-directions of the beam are given as follows:

$$\varepsilon(\mathrm{x},\mathrm{z}) = -\mathrm{z}\frac{\mathrm{F_z}L^2}{8bD_{11}}\left(\frac{\partial^2 \upsilon(\mathrm{x})}{\partial x^2} + 2S_\theta\frac{\partial^2 \varphi_\theta(\mathrm{x})}{\partial x^2}\right) + \varphi(\mathrm{z})\frac{\mathrm{F_z}}{\mathrm{b}D^a{}_{11}}(\frac{L^2}{2}\,S_\theta\,\frac{\partial^2 \varphi_\theta(\mathrm{x})}{\partial x^2}) \quad (39)$$

Furthermore, the transverse shear deformation is written as:

$$\gamma^0{}_{\mathrm{xz}}(x,z) = \frac{\partial\,\varphi(z)}{\partial z}\frac{(\mathrm{F_z}=qL)L^2}{4\mathrm{b}D^a{}_{11}}S_\theta\frac{\partial \varphi_\theta(\mathrm{x})}{\partial x} \quad (40)$$

DISCUSSION OF NUMERICAL RESULTS

Numerical characteristics and discussion of Case1:

The geometrical and mechanical properties of the studied beam are as follows:

Length L=1m; Width b=0,1m; Height h=0,01m; Load F_Z =1kn; E_c=151GPa; E_m=75,5GPa; υc=0.3; υ_m=0.25.

Figure 3 presents the variation of transverse max displacements predicted from the proposed analytical model of case1, it can be seen that the total displacement is greater. These distributions are not linear for homogeneous beams. We note that the influence of the material parameter is unlimited and its increase.

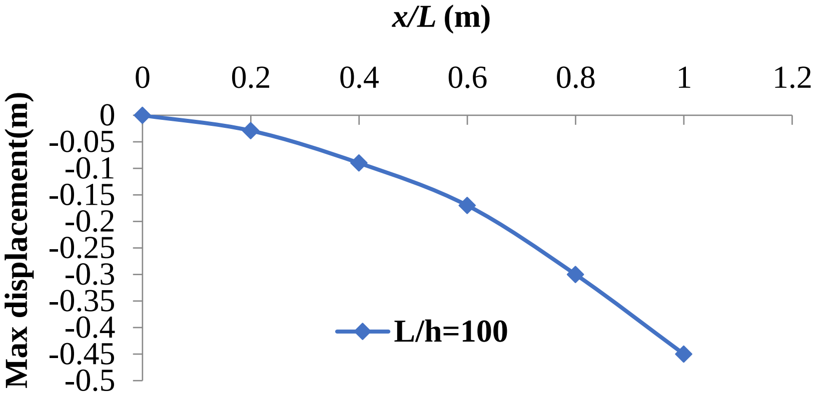

Figure 3 - Distribution of max displacement versus (*x/L*) as function as (L/h=100) for cantilever beam under concentrated load.

Figure 4 presents the variation of the transverse shear stress through the thickness of the cantilever FGM beam under concentrated load, and shows a cubic distribution of transverse shear stresses using higher-order theories (PDSBT).

ACTA SCIENTIAE

ISSN: 2178-7727

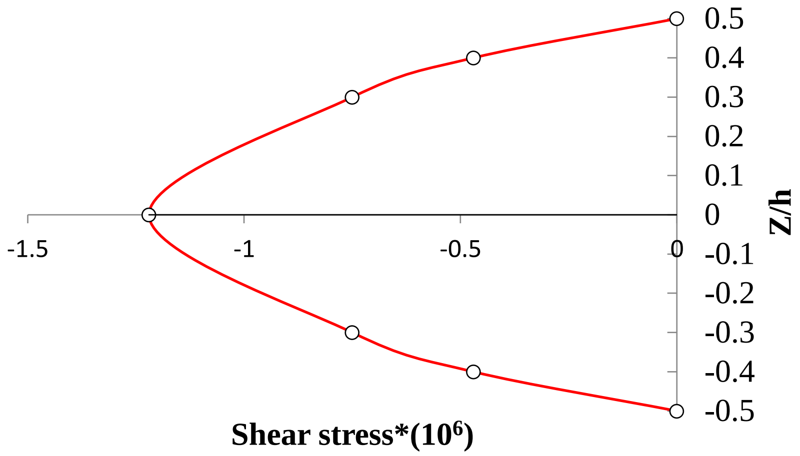

Figure 4 - Distribution of shear stress versus (*Z/h*) through the thickness of the beam for(L/h=100) for cantilever beam under concentrated load.

Numerical characteristics and discussion of Case2:

The geometrical and mechanical properties of the cantilever beam subjected to distributed uniform load are as follows: Length L=1m; Height h=0.01m; Width b=0.001m; Load q=1Pa; υ=0.25; E_c=10000GPa; E_m=1000GPa; υ=0.25; L/h=100. Figure 5 presents the variation of transverse max displacements predicted from the proposed analytical model of case2. It can be observed a max displacement curve through the thickness exhibit a nonlinear distribution for higher-order theories (PDSBT) used.

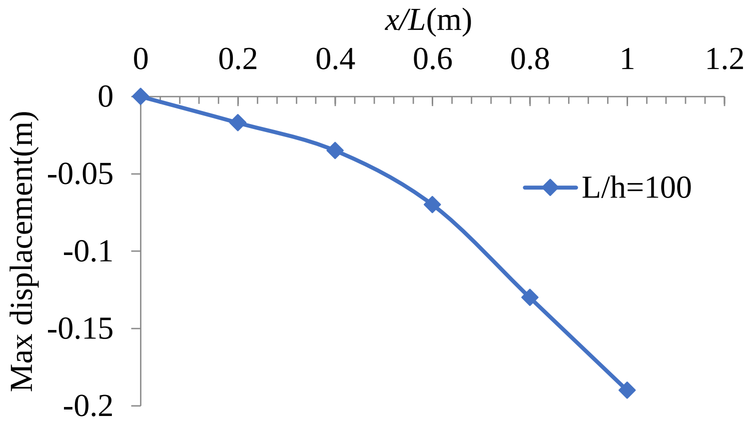

Figure 5 - Distribution max of displacement versus (*x/L*) as function as (L/h=100) for cantilever beam with uniformly distributed load.

Figure 6 presents the variation of the transverse shear stress through the thickness of the cantilever beam under uniform load. It can be noted the maximum value of the transverse shear stress predicted from the proposed function is about 0.75×10^5.

ACTA SCIENTIAE

ISSN: 2178-7727

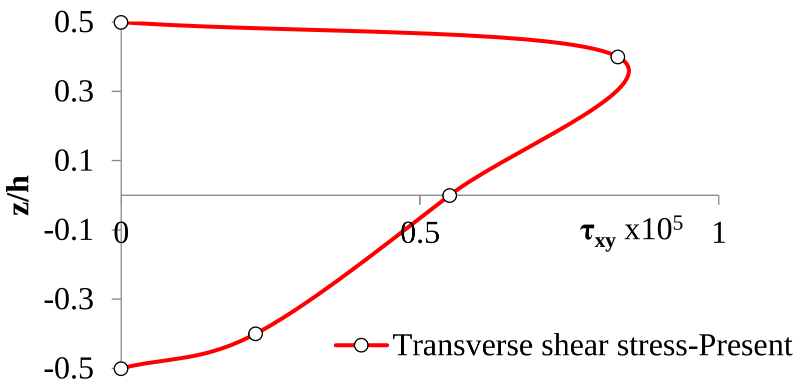

Figure 6 - Comparison of transverse shear stress variation (τ_{xz}) versus z/h of the beam for L/h=100

CONCLUSION

The Euler-Bernoulli's theory leads to an expression for simple bending without shear. For Timoshenko's theory is introduced that allows for the shear to be taken into account as linear shape functions. Finally, Higher-order shear strains new theories are used which include nonlinear shape functions and stress nullities at the bottom and top faces of the section through the beam thickness. A new analytical model is proposed taking into account shear strains in displacement fields for cantilever FGM beams under bending. The results in terms of displacement fields including shear strains and section rotation, strains and stresses, are presented. In fact, the results obtained in this study allow drawing the following conclusions:

1. Displacement assumed to be varied in increase when the ratio (L/h) increases and vice versa until failure of section under loading. The proposed mathematical model taking into account the shear strains in the displacement field is efficient for cantilever FGM beam under bending.
2. The new polynomial shear function exhibits a good convergence with those of the other higher order shear deformation beam theories. Thus, the proposed shear function can be used by the future studies.
3. On the neutral axis, can be seen the shear stress across beam thickness is symmetric.
4. Thickness of beam increase must lead to an increase of the transverse shear stress. Furthermore, having the new polynomial shear function satisfied the stress-free boundary conditions.

Appendix-1: Case-1

$$\frac{\partial \varphi_\theta(x)}{\partial x} = \sinh \Omega_\theta x - \cosh \Omega_\theta x - 1$$

$$w(x) = \frac{F_z L^2}{3bD_{11}}\left[-\frac{1}{2L^2}x^3 + \frac{3}{2}L + \frac{6}{4L^2}\frac{{D^a}_{11}{}^2}{{A^a}_{55}D_{11}}\left[x + \frac{1}{\Omega_\theta}\left(\cosh(\Omega_\theta x) - \sinh(\Omega_\theta x) - 1\right)\right] \right]$$

$$\theta(x) = \frac{F_z L^2 {D^a}_{11}}{3bD_{11}\,{A^a}_{55}}\left[\frac{3\,{A^a}_{55}}{2{D^a}_{11}L^2}x^2 + \frac{6}{4L^2}\left(\frac{{D^a}_{11}}{D_{11}}\right)\left[1 - \cosh(\Omega_\theta x) + \sinh(\Omega_\theta x)\right]\right]$$

Appendix-2: Case-2

$$\frac{\partial \varphi_\theta(x)}{\partial x} = \frac{\cosh(\Omega_\theta x)}{\cosh(\Omega_\theta L)} - \frac{\sinh \Omega_\theta (L - x)}{\Omega_\theta\, L \cosh(L\Omega_\theta)} - \frac{x}{L}$$

$$w(x) = \frac{(q)L^2}{8bD_{11}}\left[\frac{x^4}{3L^2} - \frac{4}{3L}x^3 + 2x^2 + \frac{({D^a}_{11})^2}{D_{11}\,{A^a}_{55}}\left(1 - \frac{x^2}{L^2} + \frac{2\,\Omega_\theta L\left[\sinh(\Omega_\theta L) - \sinh(\Omega_\theta x)\right]}{(L\Omega_\theta)^2\cosh(L\Omega_\theta)} + \frac{2\cosh\Omega_\theta(L - x)}{(L\Omega_\theta)^2\cosh(L\Omega_\theta)}\right)\right]$$

$$\theta(x) = \frac{qL^2 {D^a}_{11}}{8bD_{11}\,{A^a}_{55}}\left[\frac{4\,{A^a}_{55}}{3{D^a}_{11}L^2}x^3 - \frac{4\,{A^a}_{55}}{L{D^a}_{11}}x^2 + \frac{4\,{A^a}_{55}}{{D^a}_{11}}x + \left(\frac{{D^a}_{11}}{D_{11}}\right)\left(\frac{\cosh(\Omega_\theta x)}{\cosh(\Omega_\theta L)} - \frac{\sinh\Omega_\theta(L - x)}{\Omega_\theta\, L\cosh(L\Omega_\theta)} - \frac{x}{L}\right)\right]$$

ISSN: 2178-7727

Acknowledgements

Thanks to:

- General Directorate of Scientific Research and Technological Developments (DGRSDT), MESRS, Algeria.
- University Mohamed Khider of Biskra, Algeria.

REFERENCES

1. Akbas S., D. (2018). Large deflection analysis of a fiber reinforced composite beam, Steel and Composite Structures. 27, 567-576. DOI: https://doi.org/10.12989/scs.2018.27.5.567
2. Aydogdu, M., V. Taskin. (2007). Free vibration analysis of functionally graded beams with simply supported edges. Materials and Design.28, 1651-1656.
3. Benatta, M.A., Mechab, I., Tounsi, A., Adda Bedia, E.A. (2008). Static analysis of functionally graded short beams including warping and shear deformation effects. J Computational Materials. 44, 765-773. https://doi.org/10.1016/j.commatsci.2008.05.020
4. Benatta, M.A., Tounsi, A., Mechab, I., Bouiadjra, M.B. (2009). Mathematical solution for bending of short hybrid composite beams with variable fibers spacing. Applied Mathematics and Computation. 212, 337-348. https://doi.org/10.1016/j.amc.2009.02.030
5. Bouremana M., Houari M.S.A., Tounsi A., Kaci A., Bedia E.A.A. (2013). A new first shear deformation beam theory based on neutral surface position for functionally graded beams. Steel and Composite Structures. 15, 467-479. DOI: 10.12989/scs.2013.15.5.467
6. Dahake A. G., Yuwaraj M. Ghugal, (2012). Flexure of Thick Simply Supported Beam Using Trigonometric Shear Deformation Theory. International Journal of Scientific and Research Publications.
7. Gandhe G. R., P. M. Pancake, D. H. Tupe , P. G. Taur(2018) . Higher Order Computational Method for Static Flexural Analysis of thick beam. Procedia Manufacturing. 20, 493–498. https://doi.org/10.1016/j.promfg.2018.02.073
8. Ghugal Y. M., Sharma, R. (2009). A Hyperbolic Shear Deformation Theory for Flexure and Vibration of Thick Isotropic Beams. International Journal of Computational Methods. 6, 585-604. doi:10.1142/S0219876209002017
9. Ghugal Y. M. and Sharma, R., (2011) . A Refined Shear Deformation Theory for Flexure of Thick Beams, Latin American Journal of Solids and Structures. 8 ,183-193.
10. Guenfoud, H., Ziou, H., Himeur, M., Guenfoud, H. (2016). Analyses of a composite functionally graded material beam with a new transverse shear deformation function. Journal applied engineering science technologie,2, 105-113.
11. Huang Y., Li X.F.A. (2010). New approach for free vibration of axially functionally graded beams with non-uniform cross-section, Journal of Sound and Vibration. 329, 2291–2303. DOI: 10.1016/j.jsv.2009.12.029
12. Reddy, JN. (2000). Analysis of functionally graded plates. International journal for numerical methods in engineering.47, 663- 84.
13. Şimşek, M. (2009). Static analysis of a functionally graded beam under a uniformly distributed load by Ritz method. International Journal of Engineering and Applied Sciences, 1, 1-11.
14. Zaoui, F. Z., Hanifi, A., Younsi, A., Meradjah, M, Tounsi., A., Ouinas. D. (2017). Free vibration analysis of functionally graded beams using a higher-order shear deformation theory. Mathematical Modelling of Engineering Problems. 4, 7-12. DOI: 10.18280/mmep.040102
15. Zhong, Z.,Yu, T.(2007). Analytical solution of cantilever functionally graded beams. Composites science technology Journal. 67, 481–488. https://doi.org/10.1016/j.compscitech.2006.08.023
16. Ziou, H., Guenfoud., M., Guenfoud, H.(2021). A Simple Higher-order Shear Deformation Theory for Static Bending Analysis of Functionally Graded Beams. Jordan Journal of Civil Engineering. 15, 209-224.